Binary Trees

Lecture 13

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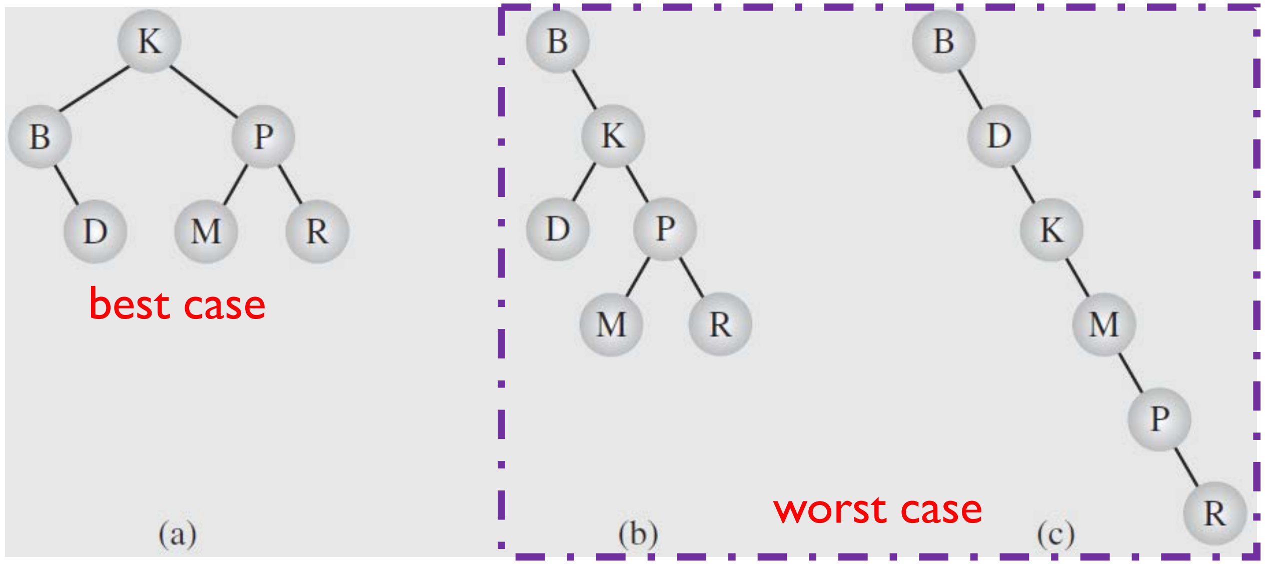
Adapted partially from Data Structures and Algorithms in Java, M.T. Goodrich, R.Tamassia and M. H. Goldwasser, Sixth Edition, Wiley; Data Structures and Algorithms in C++, Adam Drozdek, 4th Edition, Cengage Learning

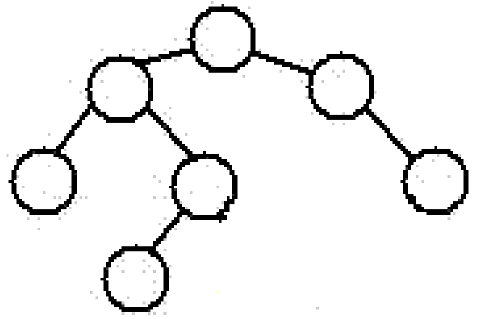
Balancing a Tree

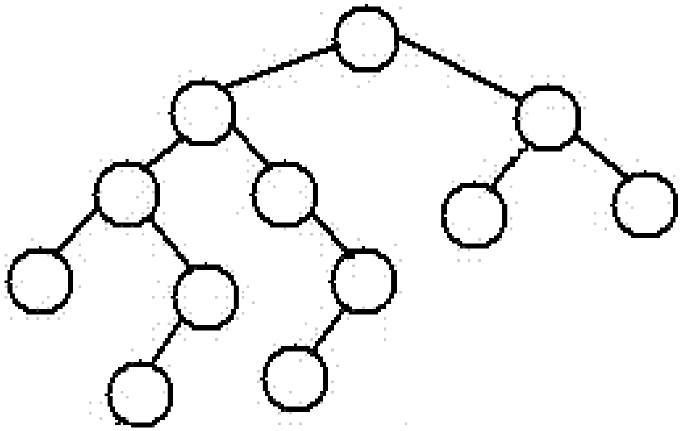
- In favor of trees,
  - represent hierarchical data particularly well
  - searching trees is much faster than searching lists
    - depends on the **structure of the tree**
    - e.g., skewed trees search no better than linear lists



unsymmetrical: objects are not distributed evenly

Balancing a Tree

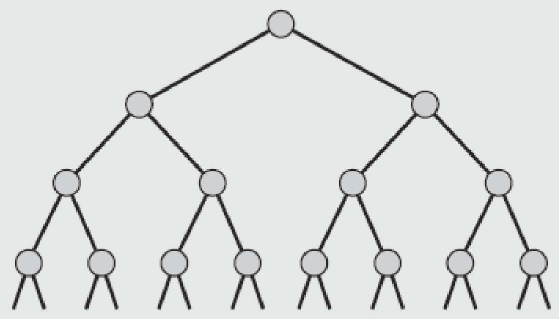

A height-balanced Tree


Not a height-balanced tree

- A binary tree is ***height balanced*** (or simply, ***balanced***)
  - if the ***difference*** in **height** of both ***subtrees*** of ***any node*** in the tree is **zero or one**

Balancing a Tree (cont.)

- Maximum number of nodes that can be stored in binary trees of different heights:



Height	Nodes at One Level	Nodes at All Levels
1	$2^0 = 1$	$1 = 2^1 - 1$
2	$2^1 = 2$	$3 = 2^2 - 1$
3	$2^2 = 4$	$7 = 2^3 - 1$
4	$2^3 = 8$	$15 = 2^4 - 1$
⋮		
11	$2^{10} = 1{,}024$	$2{,}047 = 2^{11} - 1$
⋮		
14	$2^{13} = 8{,}192$	$16{,}383 = 2^{14} - 1$
⋮		
h	2^{h-1}	$n = 2^h - 1$
⋮		

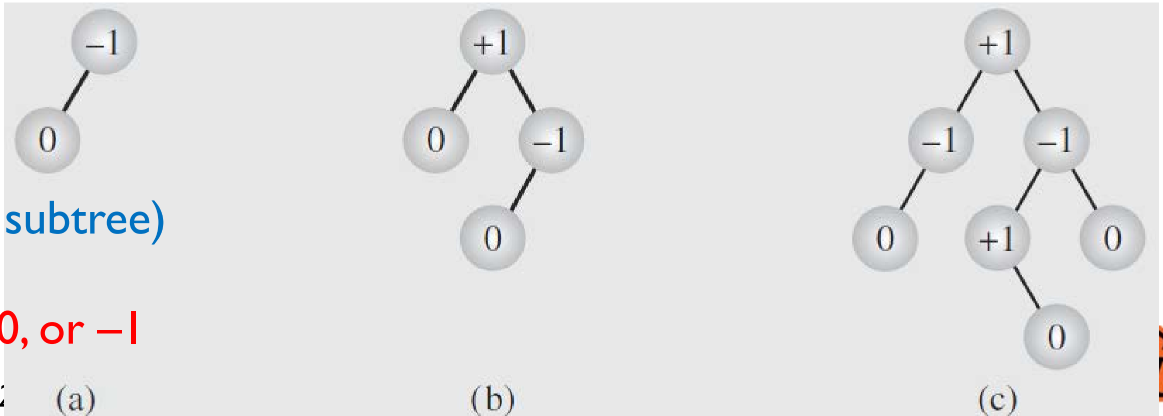
Q: if there are10,000 elements stored in a perfectly balanced tree, what is the height of tree?

AVL Trees

- Rebalancing can be performed locally,
  - if the *insertions* or *deletions* impact only a **portion** of the tree
- An ***AVL tree*** (also called an ***admissible tree***)
  - the *height* of the *left* and *right subtrees* of *every node* differ by **at most one**
  - the ***balance factors***,
    - the difference between the height of the right and left subtrees and should be +1, 0, or -1

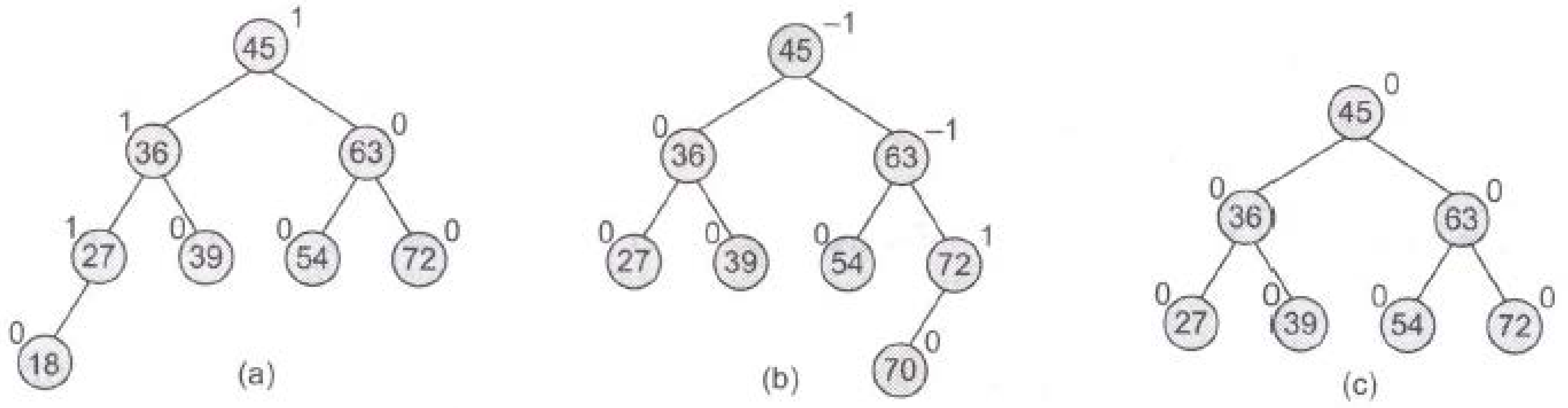
balance factor =
height (right subtree) – height (left subtree)

all balance factors should be +1, 0, or –1



AVL Trees (cont.)

- Example of AVL trees,
  - Here, balance factor = height (**<u>left</u>** subtree) – height (**<u>right</u>** subtree)

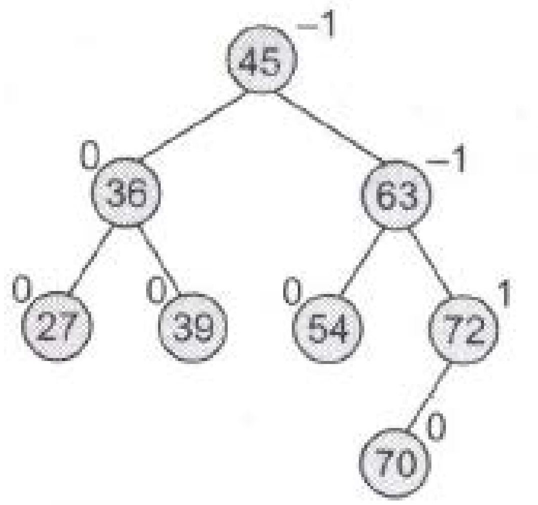


- Searching for a node
  - exactly the same way with a binary search tree

AVL Trees (cont.)

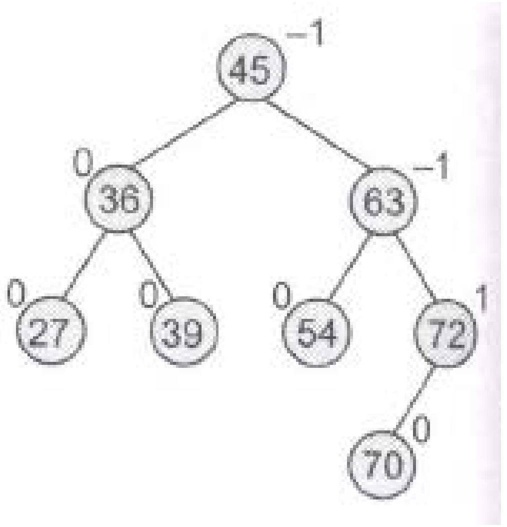
- Inserting a new node
  - if not disturb the **balance factor**
    - don’t do anything
    - e.g., inserting 30

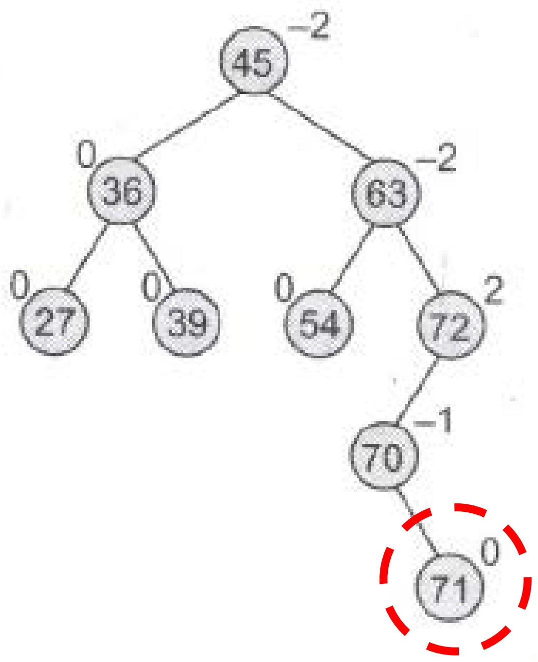
not disturbing: the balance factor of any node in an AVL tree does not become less than –1 or greater than 1.



AVL Trees (cont.)

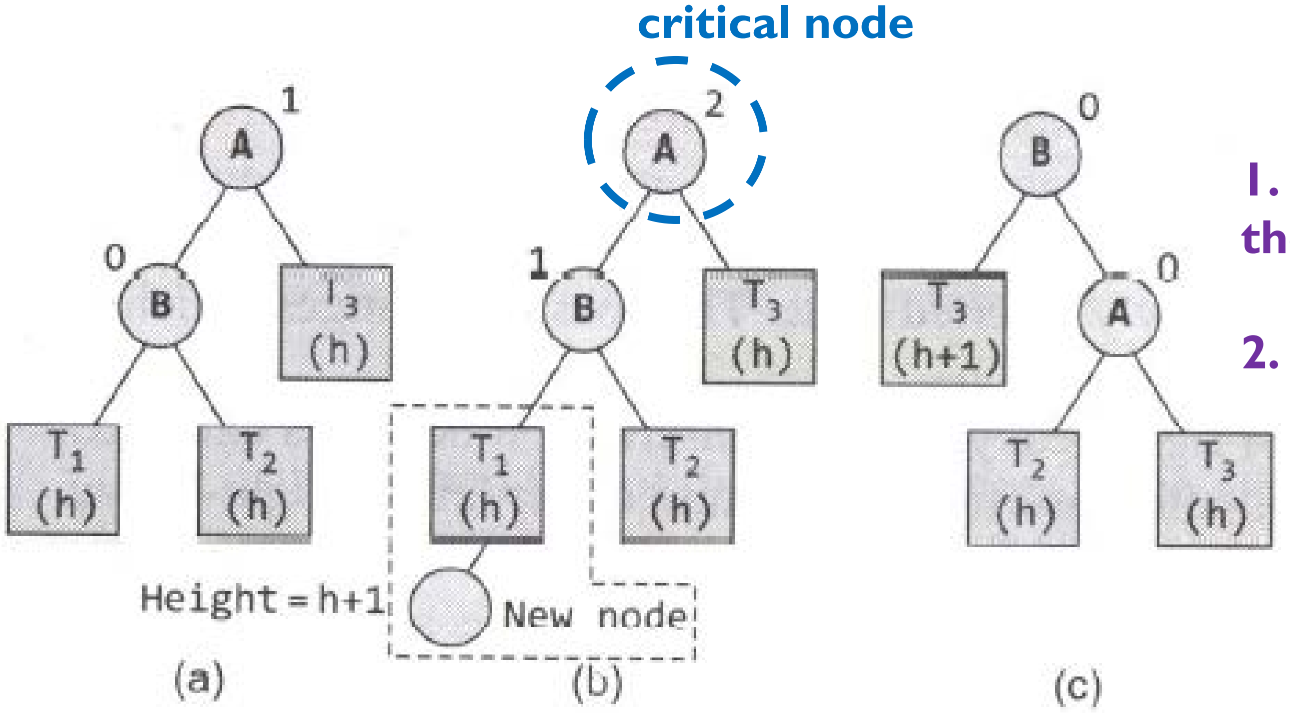
- Inserting a new node (cont.)
  - if disturbing the ***balance factor***
    - single rotation (LL & RR)
    - double rotation (LR & RL)
    - e.g., inserting 71
  - **critical node**
    - the nearest ancestor node on the path from the inserted node to the root whose balance factor is neither -1, 0, nor 1.





AVL Trees (cont.)

- Inserting a new node (cont.)
  - **single rotation (LL):** inserted in the **left subtree** of the **left subtree** of the critical node

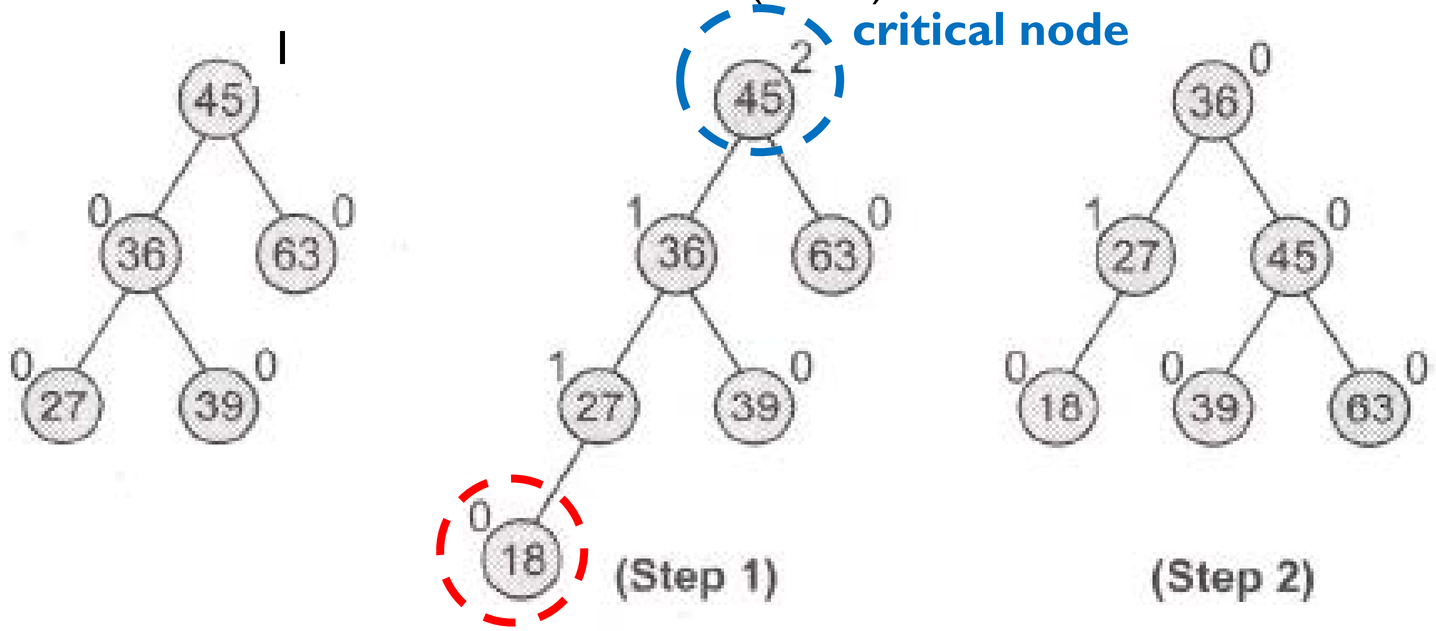


1. assign A as the parent of the right subtree of B.

2. make B as the parent of A

AVL Trees (cont.)

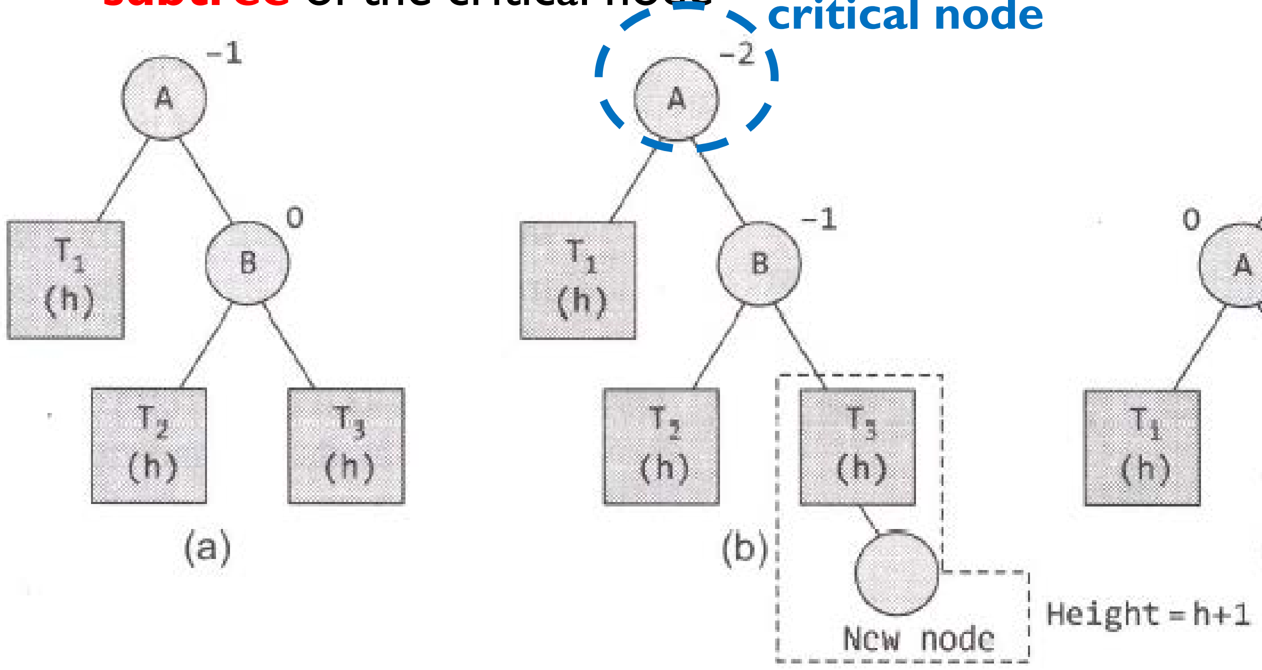
- Inserting a new node (cont.)
  - **single rotation (LL):** inserted in the **left subtree** of the **left subtree** of the critical node (cont.)



1. **assign 45 as the parent of the right subtree of 36.**
2. **make 36 as the parent of 45**

AVL Trees (cont.)

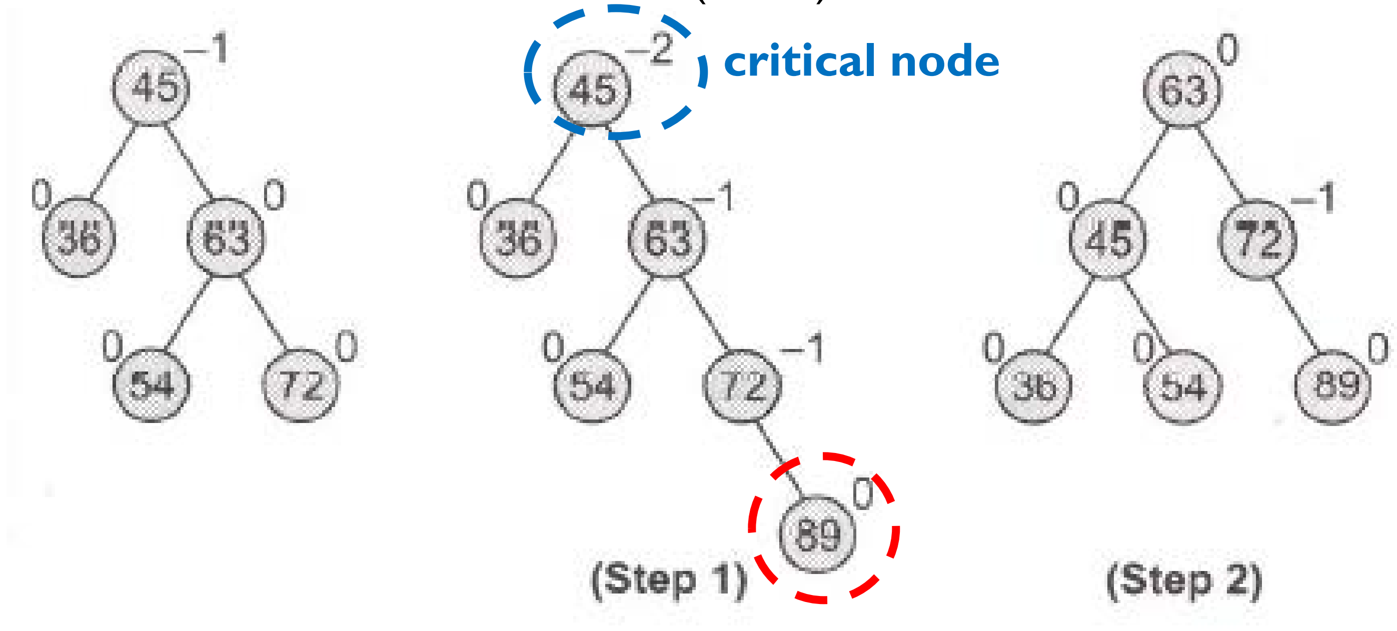
- Inserting a new node (cont.)
  - **single rotation (RR):** inserted in the **right subtree** of the **right subtree** of the critical node



1. assign A as the parent of the left subtree of B.
2. make B as the parent of A

AVL Trees (cont.)

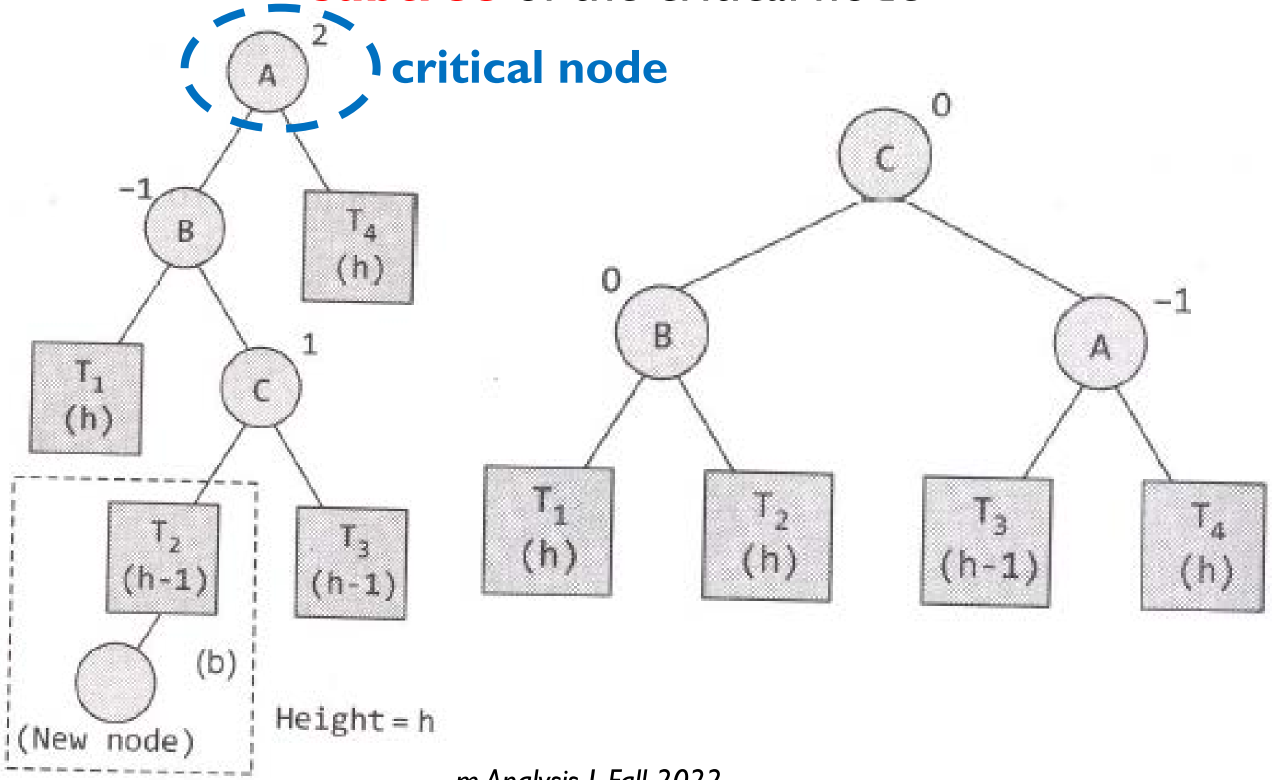
- Inserting a new node (cont.)
  - **single rotation (RR):** inserted in the **right subtree** of the **right subtree** of the critical node (cont.)



1. assign 45 as the parent of the left subtree of 63.
2. make 63 as the parent of 45

AVL Trees (cont.)

- Inserting a new node (cont.)
  - **double rotation (LR):** inserted in the **right subtree** of the **left subtree** of the critical node



1. assign B as the parent of the left subtree of C.

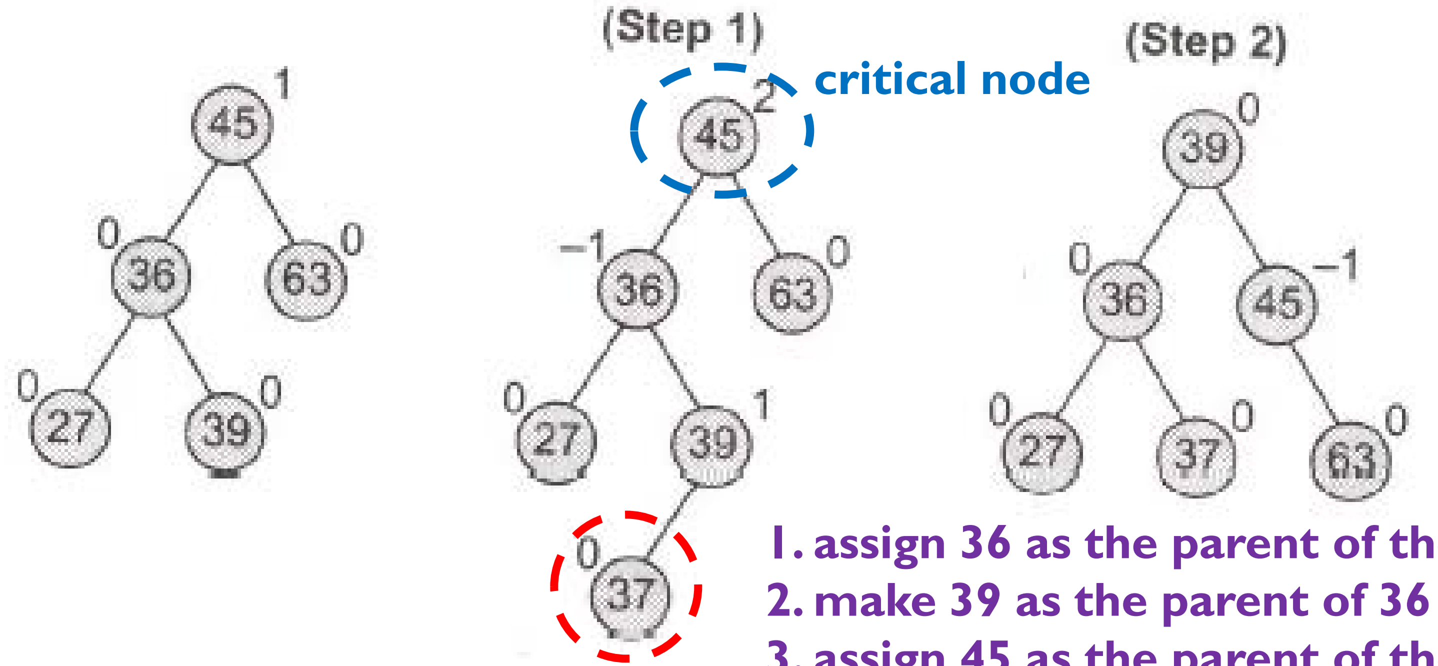
2. make C as the parent of B

3. assign A as the parent of the right subtree of C.

4. make C as the parent of A

AVL Trees (cont.)

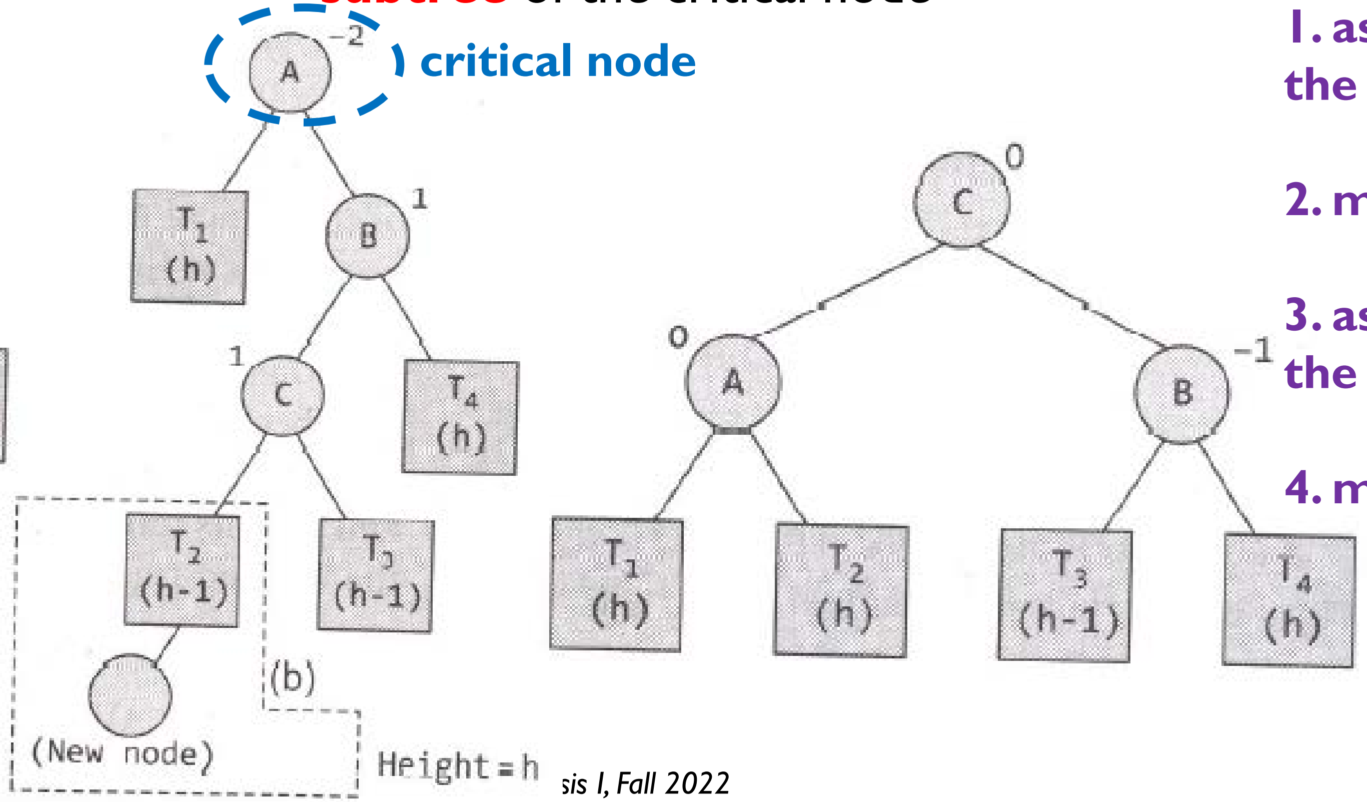
- Inserting a new node (cont.)
  - **double rotation (LR):** inserted in the **right subtree** of the **left subtree** of the critical node (cont.)



1. assign 36 as the parent of the left subtree of 39
2. make 39 as the parent of 36
3. assign 45 as the parent of the right subtree of 39
4. make 39 as the parent of 45

AVL Trees (cont.)

- Inserting a new node (cont.)
  - **double rotation (RL):** inserted in the **left subtree** of the **right subtree** of the critical node



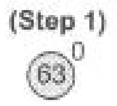
1. assign B as the parent of the right subtree of C.

2. make C as the parent of B

3. assign A as the parent of the left subtree of C.

4. make C as the parent of A

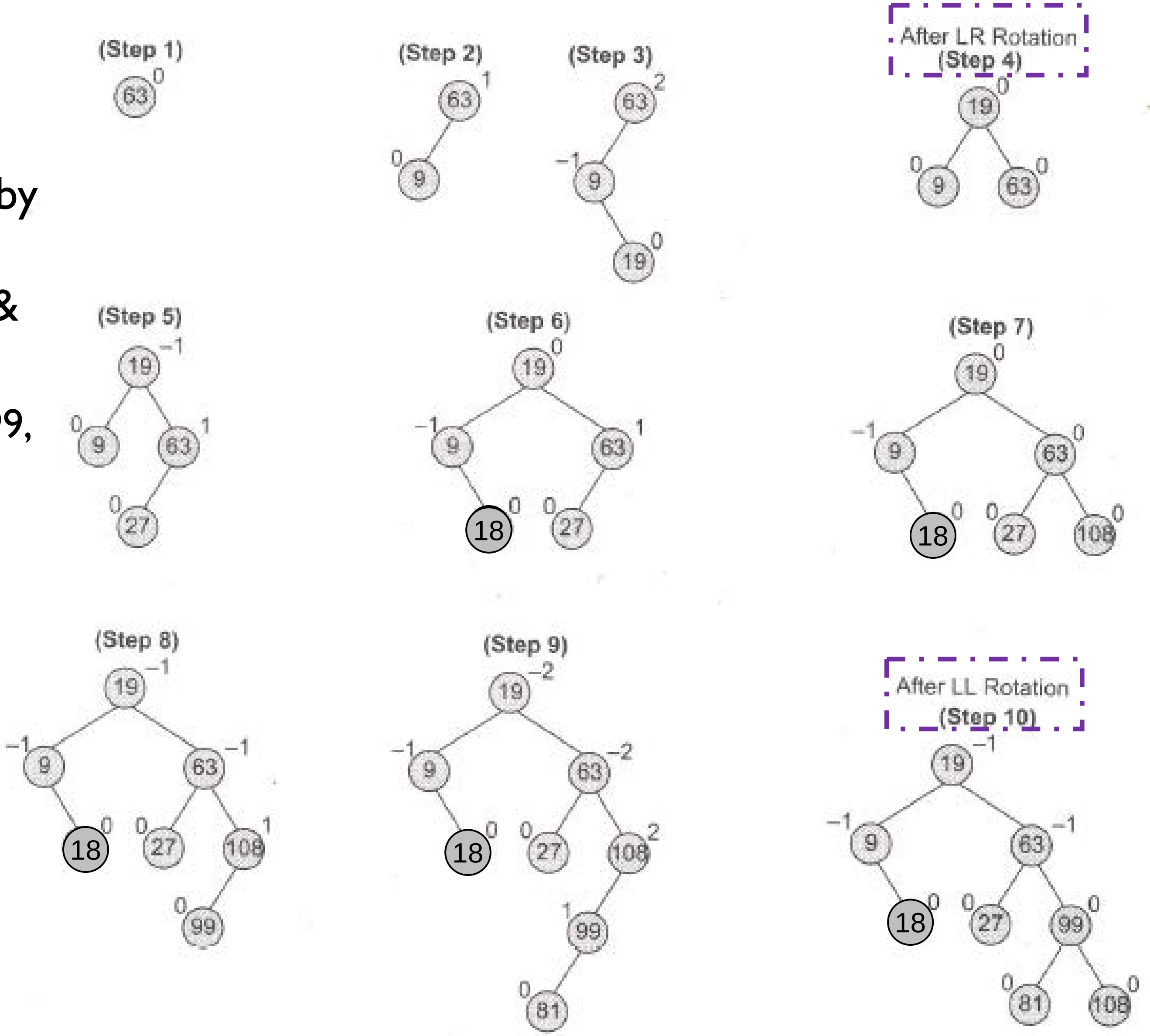
AVL Trees (cont.)



- Construct an AVL tree by inserting the following nodes in a given order & indicate any rotation
  - 63, 9, 19, 27, 18, 108, 99, 81

AVL Trees (cont.)

- Construct an AVL tree by inserting the following nodes in a given order & indicate any rotation
    - 63, 9, 19, 27, 18, 108, 99, 81

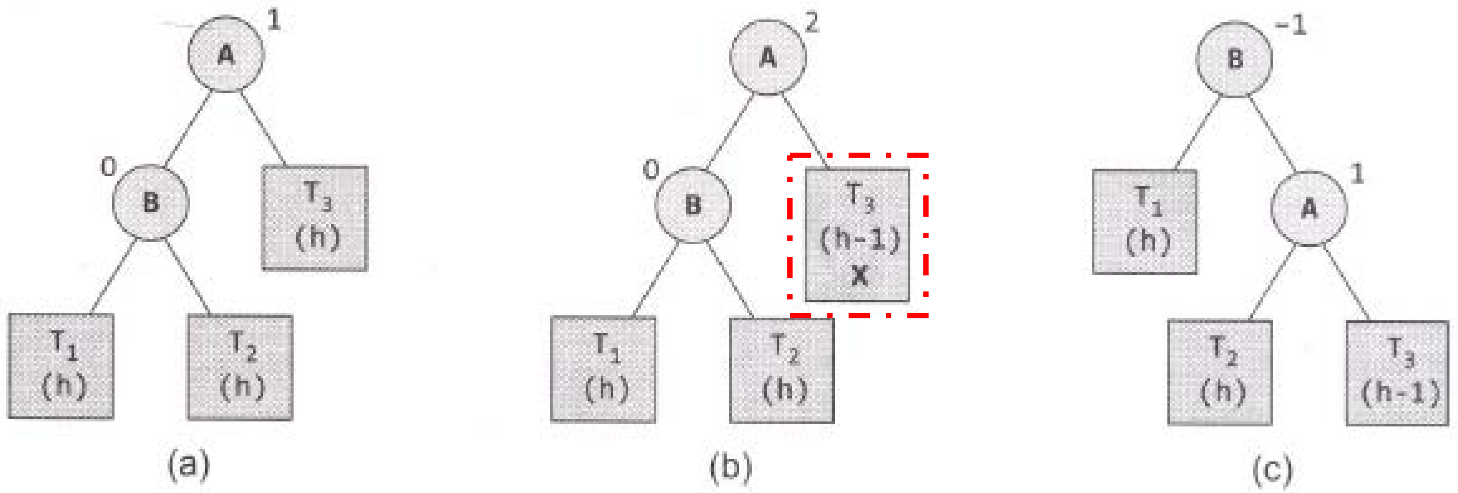


AVL Trees (cont.)

- Deleting a node
  - similar to binary search tree
  - may disturb the **balance factor**
    - **rebalance** the AVL tree
    - rotation (L & R)
- R0 rotation

1. assign A as the parent of the right subtree of B.

2. make B as the parent of A

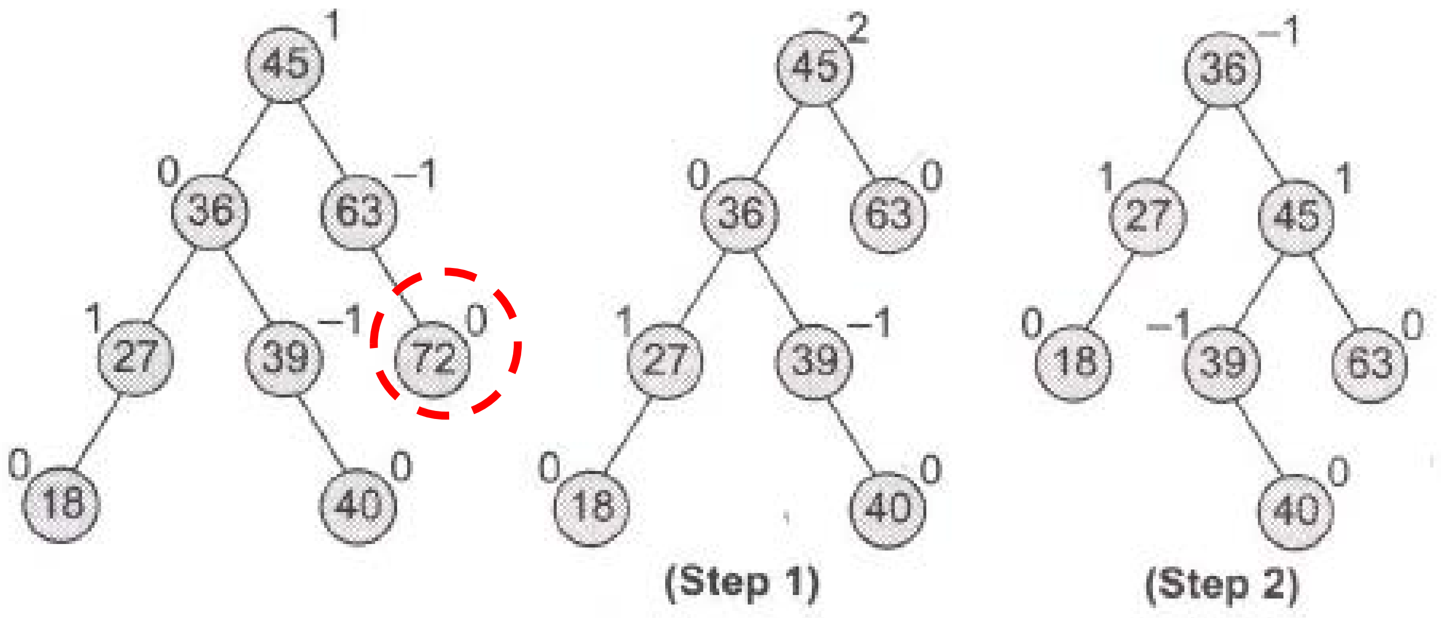


AVL Trees (cont.)

- R0 rotation: (cont.)

1. assign 45 as the parent of the right subtree of 36

2. make 36 as the parent of 45

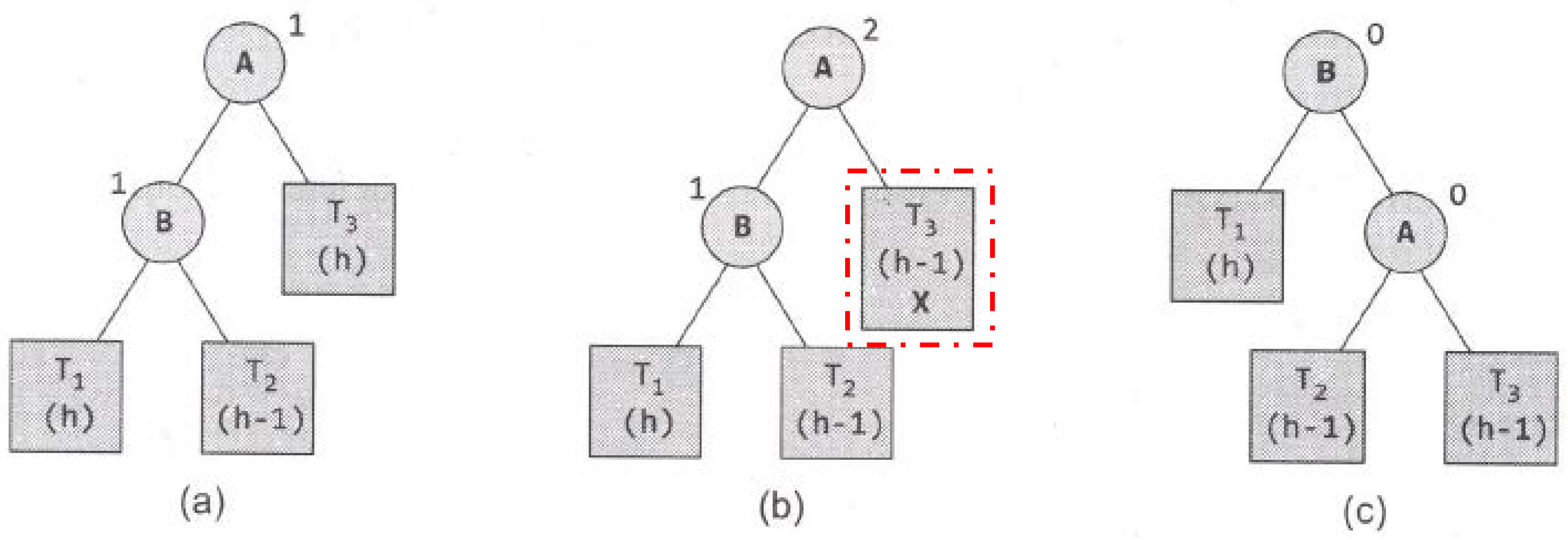


AVL Trees (cont.)

- R1 rotation:

1. assign A as the parent of the right subtree of B.

2. make B as the parent of A

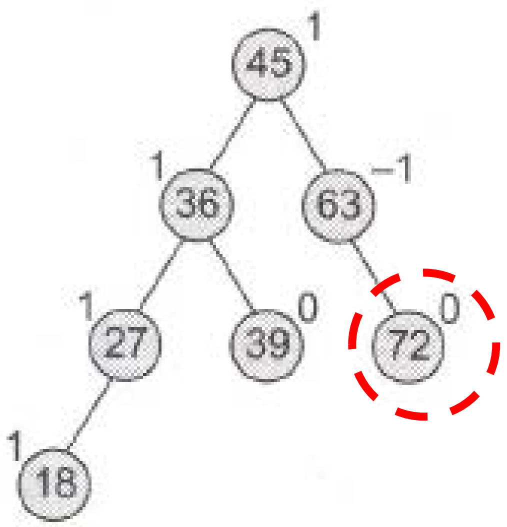


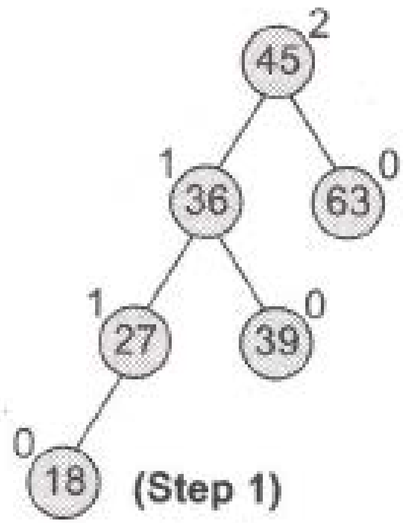
AVL Trees (cont.)

- R1 rotation: (cont.)

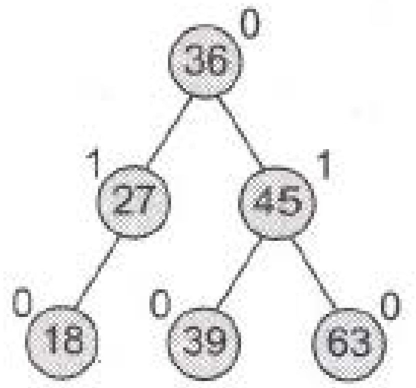
1. assign 45 as the parent of the right subtree of 36

2. make 36 as the parent of 45





(Step 1)

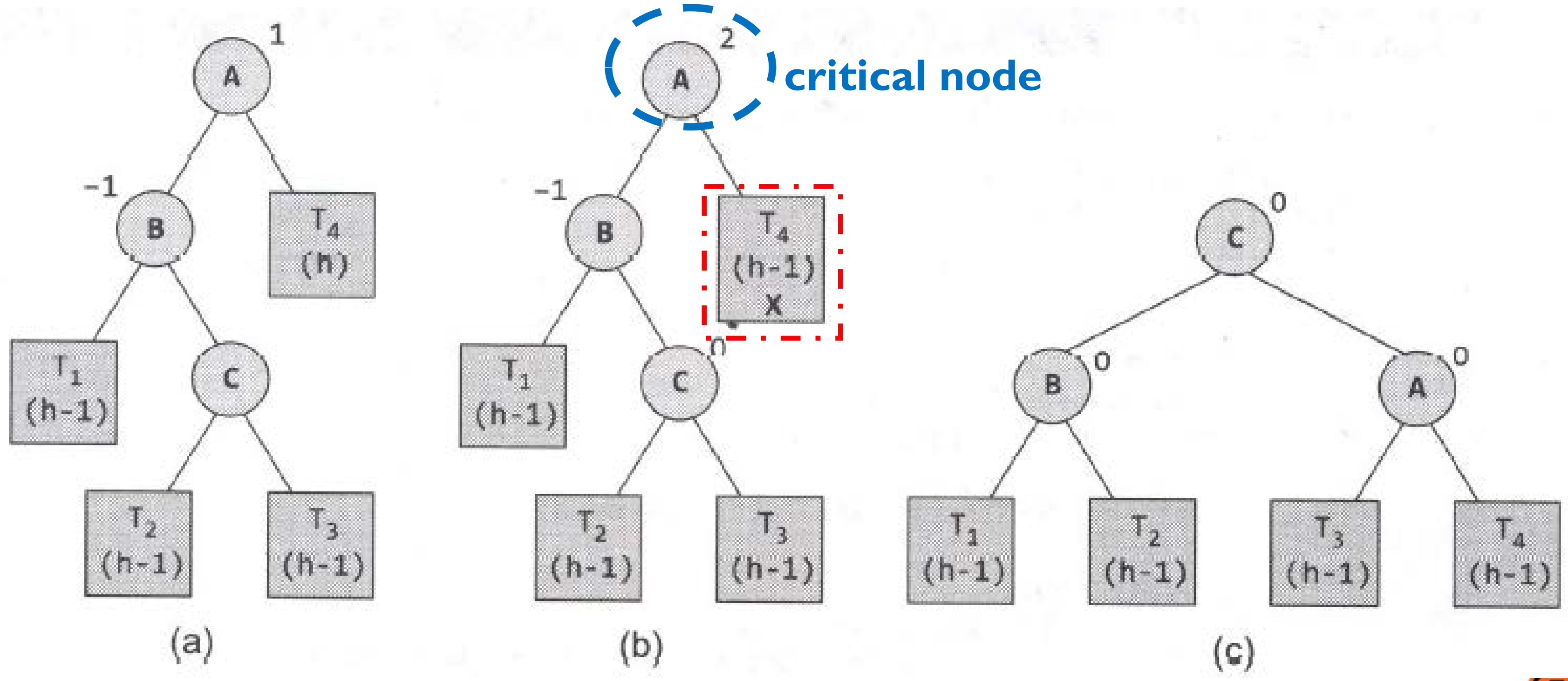


(Step 2)

AVL Trees (cont.)

- R-1 rotation:

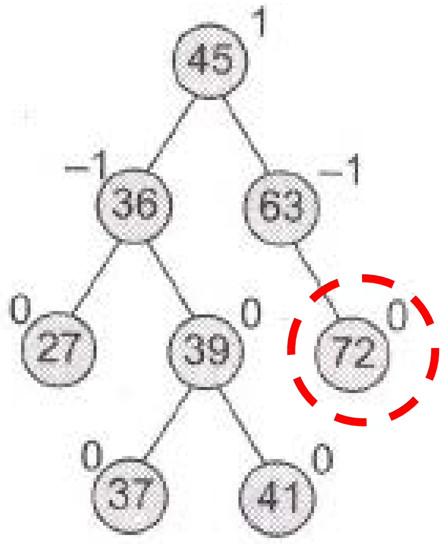
1. assign B as the parent of the left subtree of C
2. make C as the parent of B
3. assign A as the parent of the right subtree of C
4. make C as the parent of A

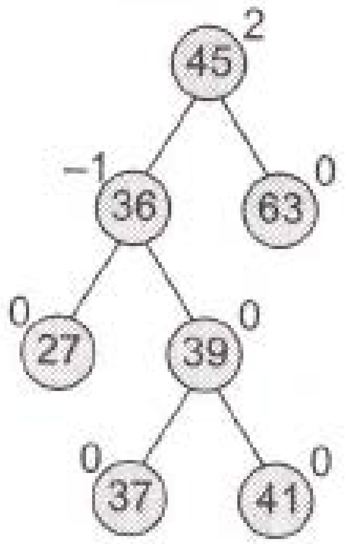


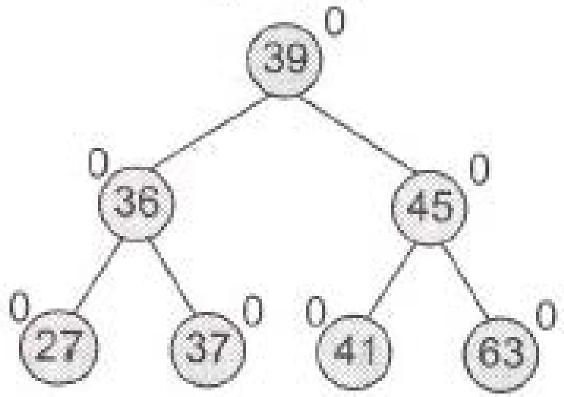
AVL Trees (cont.)

- R-1 rotation: (cont.)

1. assign 36 as the parent of the left subtree of 39
2. make 39 as the parent of 36
3. assign 45 as the parent of the right subtree of 39
4. make 39 as the parent of 45

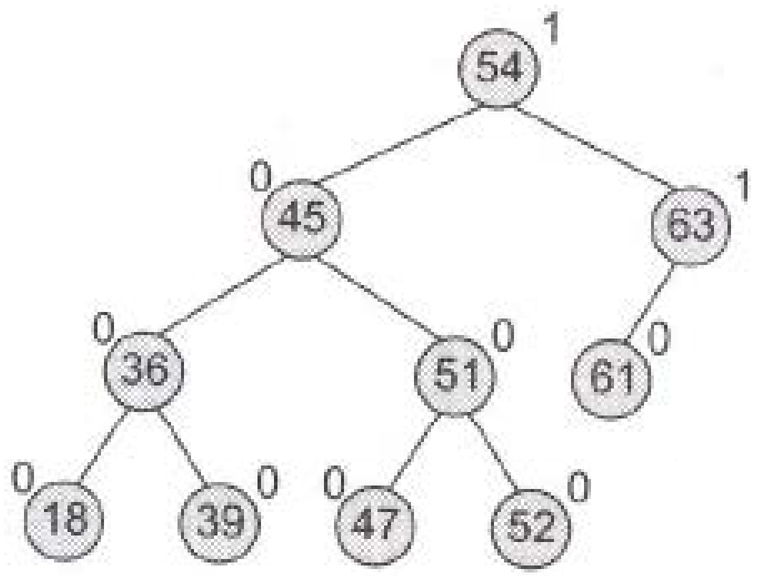


(Step 1)


(Step 2)


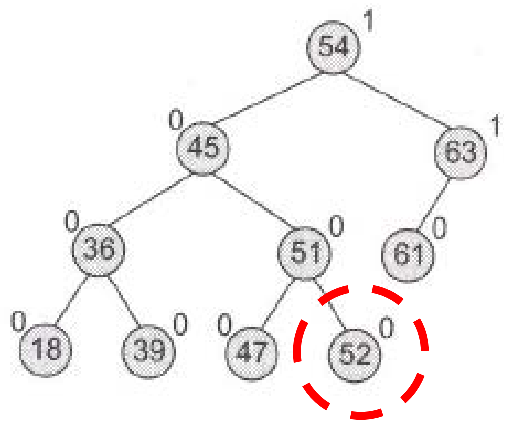
AVL Trees (cont.)

- Delete nodes 52, 36, and 61 from the AVL tree given below,



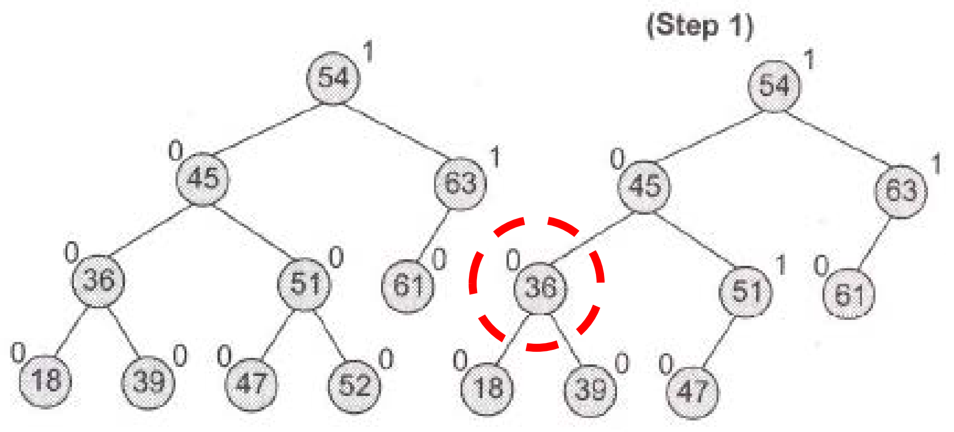
AVL Trees (cont.)

- Delete nodes 52, 36, and 61 from the AVL tree given below, (cont.)



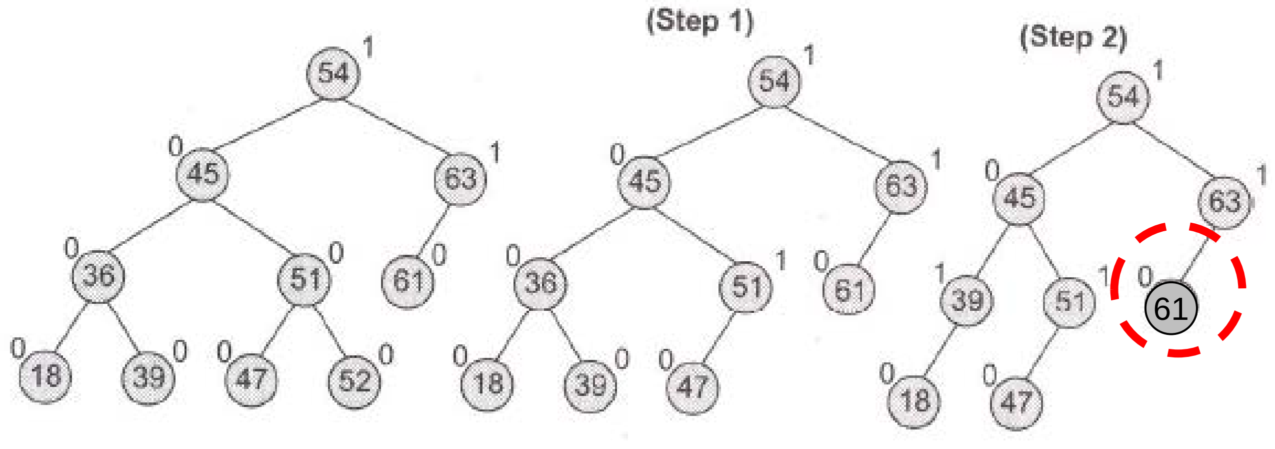
AVL Trees (cont.)

- Delete nodes 52, 36, and 61 from the AVL tree given below, (cont.)



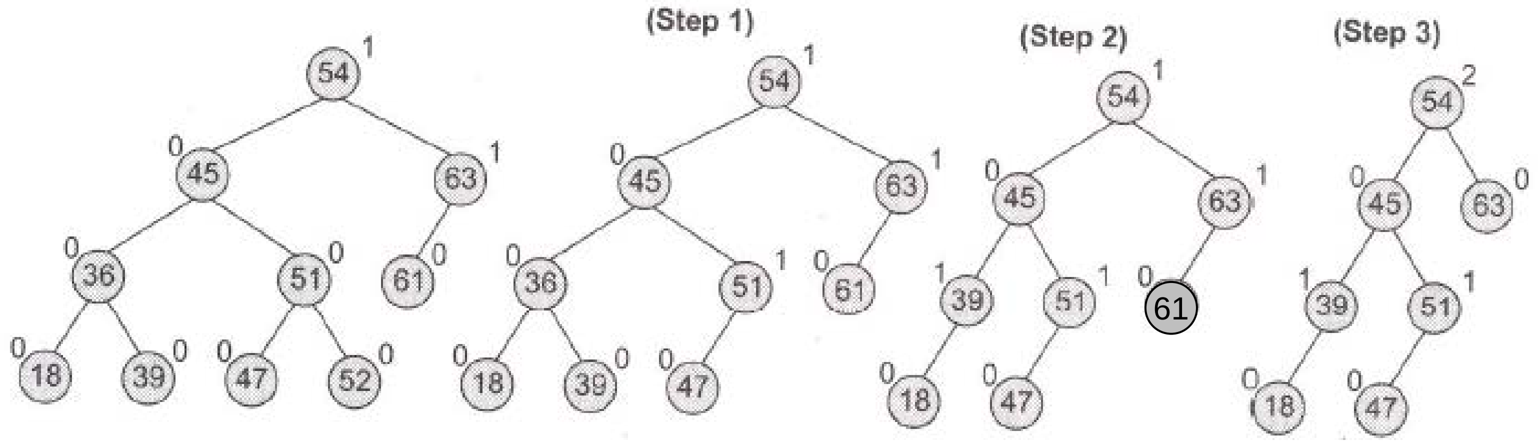
AVL Trees (cont.)

- Delete nodes 52, 36, and 61 from the AVL tree given below, (cont.)



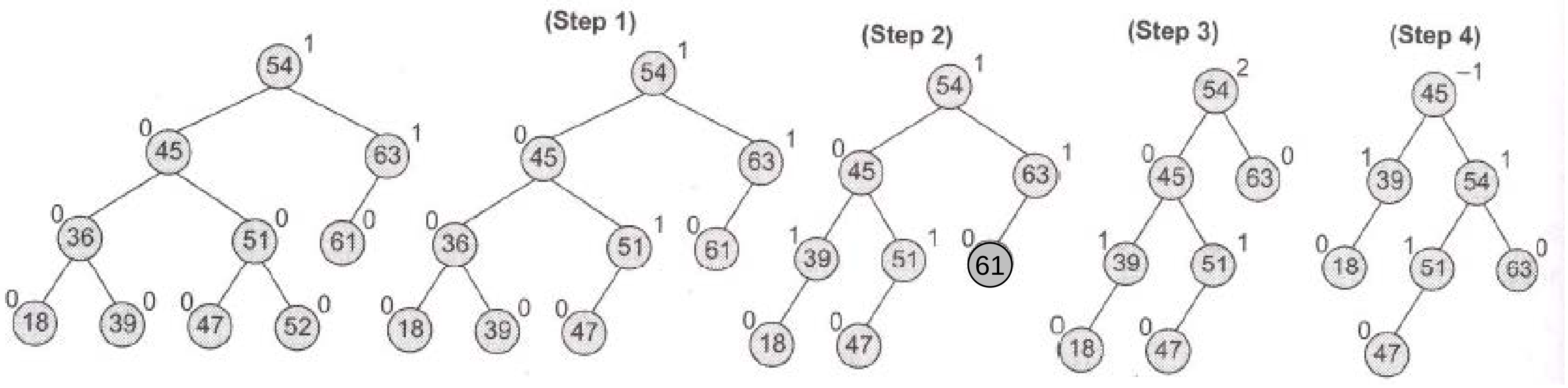
AVL Trees (cont.)

- Delete nodes 52, 36, and 61 from the AVL tree given below, (cont.)



AVL Trees (cont.)

- Delete nodes 52, 36, and 61 from the AVL tree given below, (cont.)


(Step 1)
(Step 2)
(Step 3)
(Step 4)