Graphs

Lecture 17

Instructor: Dr. Cong Pu, Ph.D.

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Adapted partially from Data Structures and Algorithms in Java, M.T. Goodrich, R.Tamassia and M. H. Goldwasser, Sixth Edition,Wiley; Data Structures and Algorithms in C++, Adam Drozdek, 4th Edition, Cengage Learning

Shortest Paths: Dijkstra Algorithm

- Finding the ***shortest path*** between two nodes,
  - the edges of the graph associated with values, e.g., distance, time, costs, amounts, etc.

- **Dijkstra's algorithm,**
  - find the **shortest path** between **source node** and **every other node**
  - if the path is **longer** than any other path from that point, it is **dropped**, and the other path is expanded
  - each vertex is visited, the new paths are started, and the vertex is then not used anymore
  - once all the vertices are visited, the algorithm is done

Dijkstra Algorith

dist[v]: shortest distance from ***s*** to ***v***

previous[v]: previous vertex in shortest path to v

Q: set of vertices

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Dijkstra (G, s)
    for each vertex v in G
        dist[v] = ∞
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    Q = G.V
    while Q is not empty
        u = node in Q with smallest dist[]
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            alt = dist[u] + dist_between(u, v)
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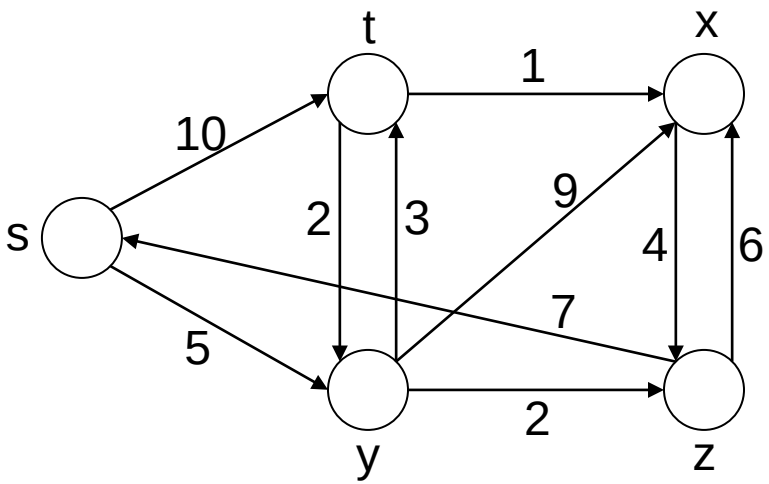


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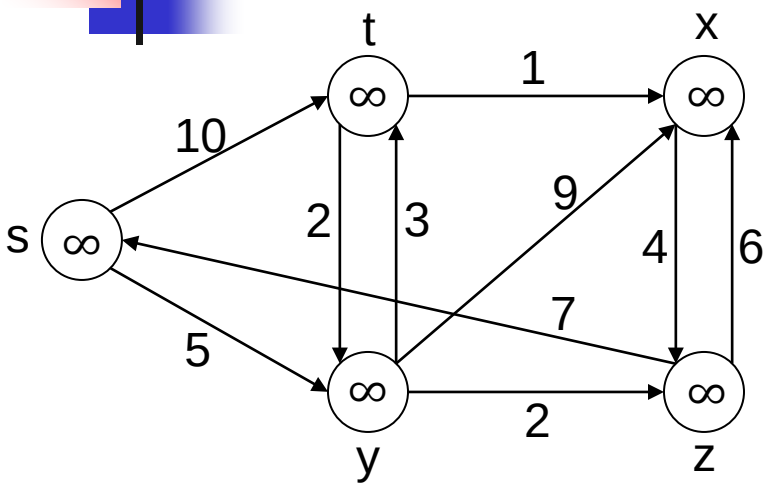


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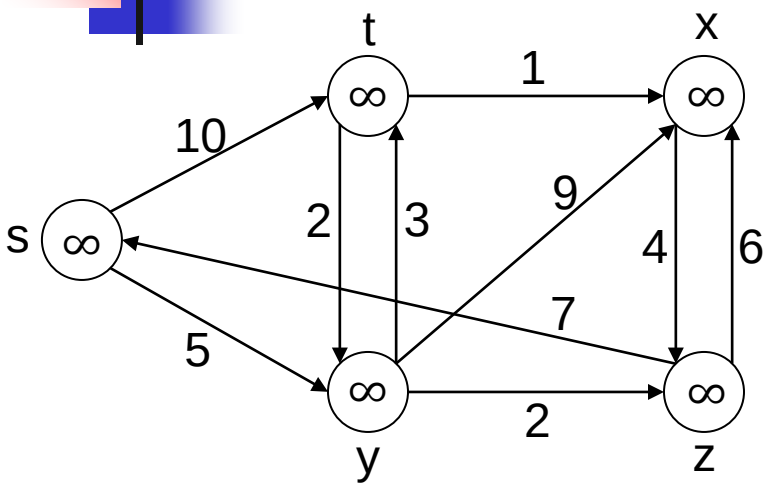


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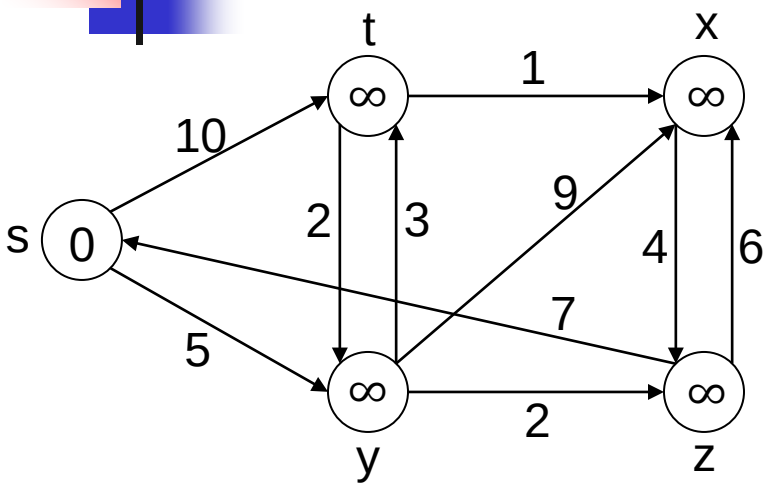


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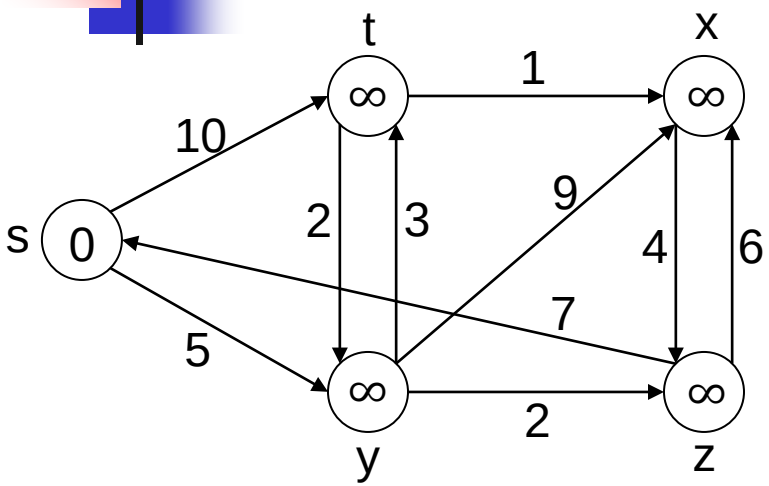

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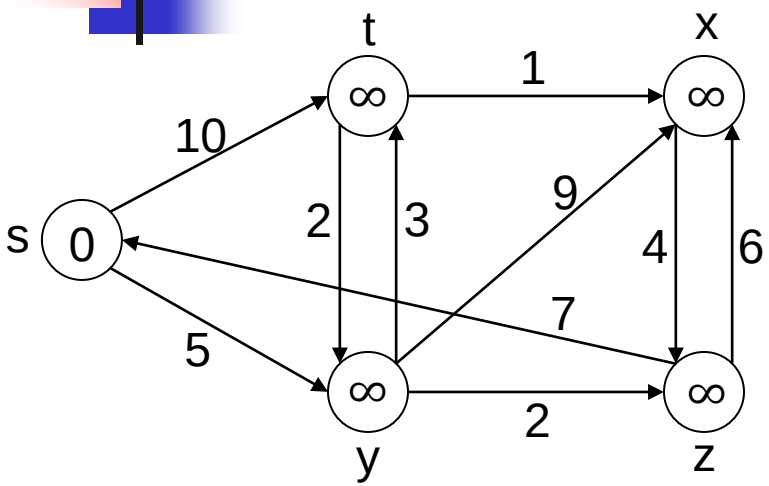

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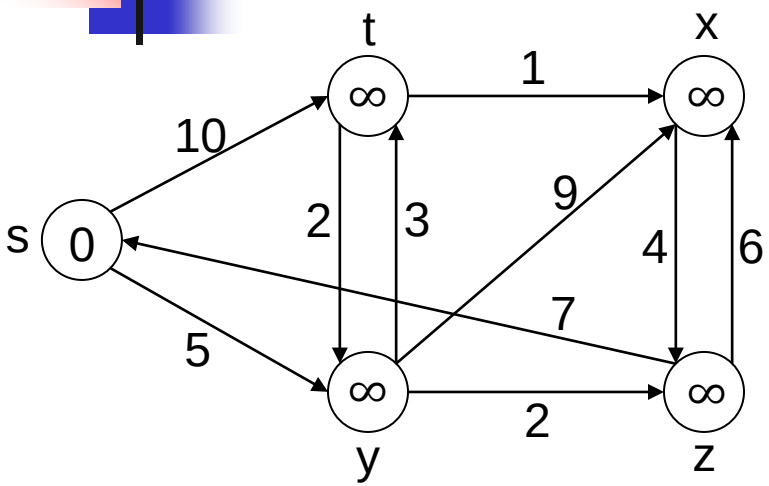

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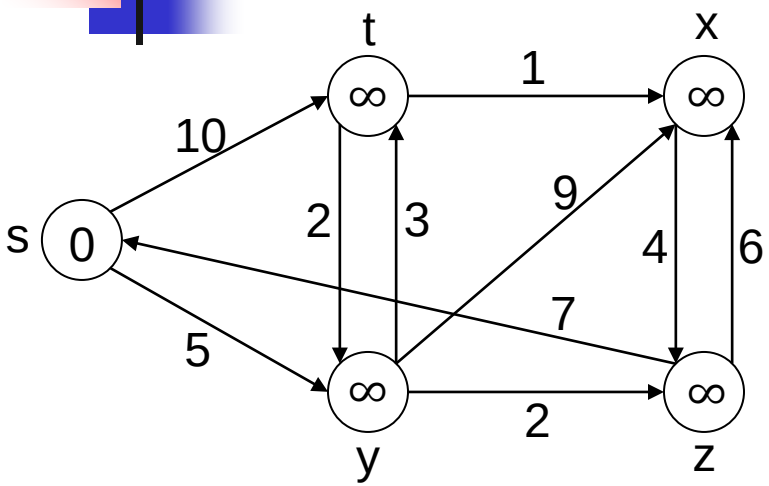

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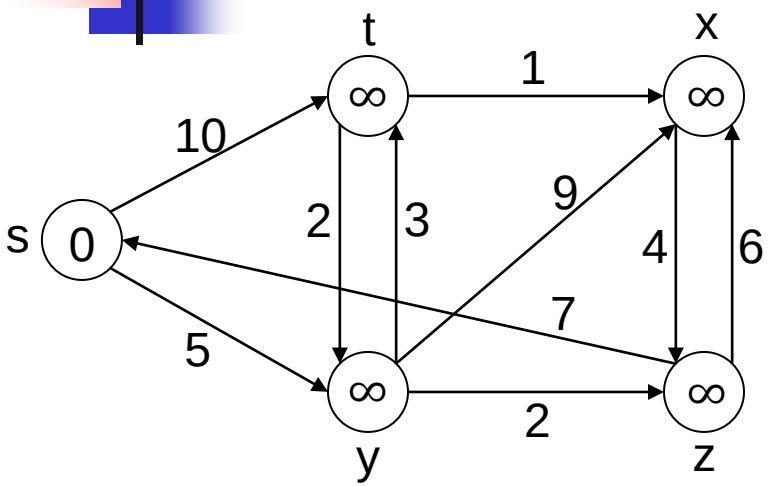

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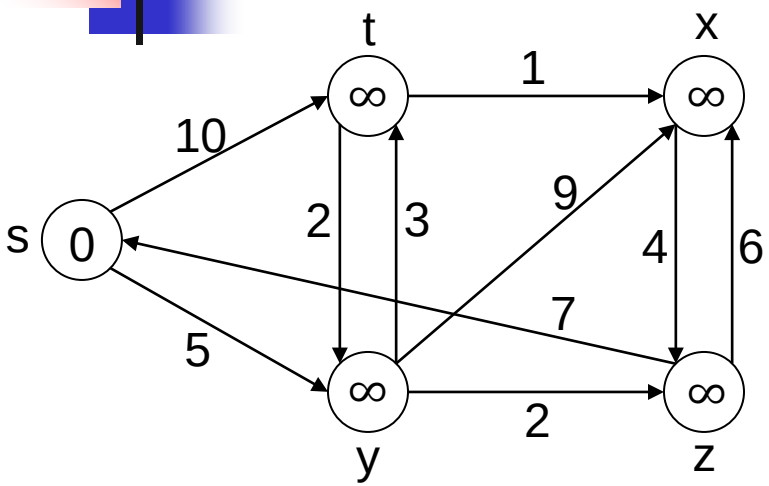

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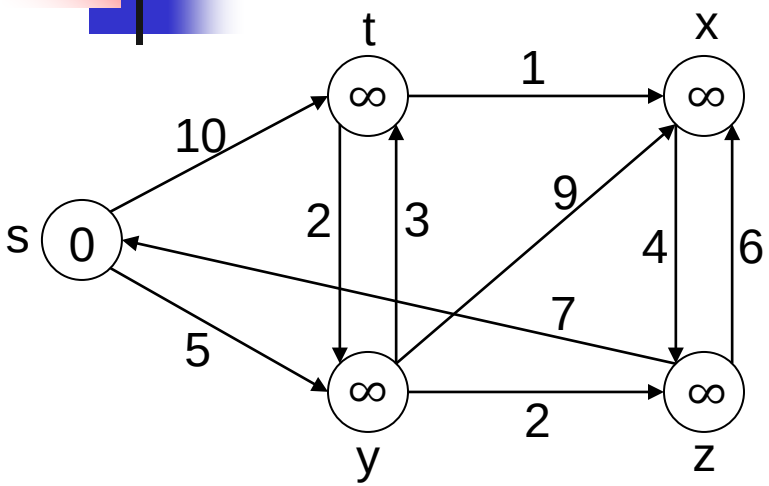

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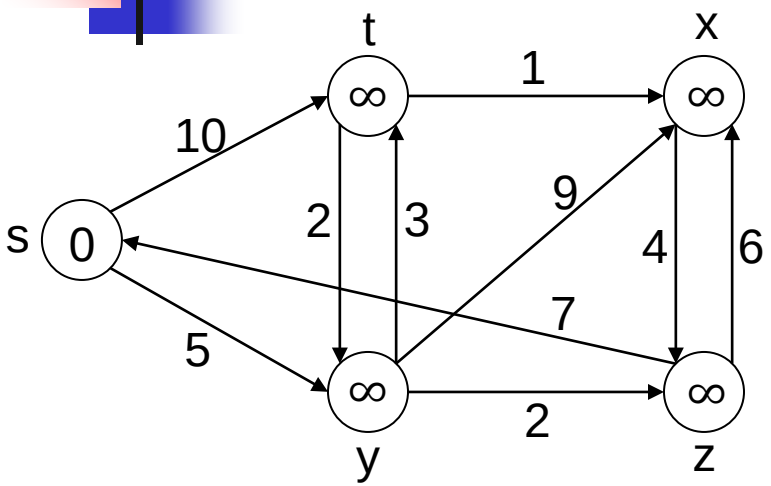

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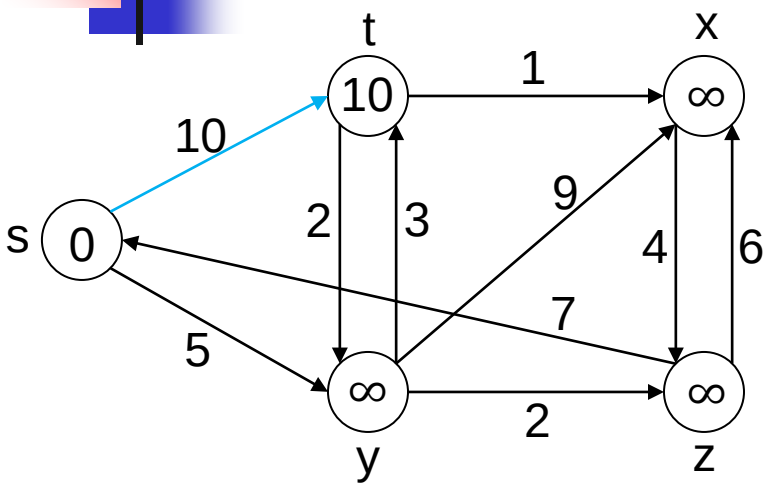

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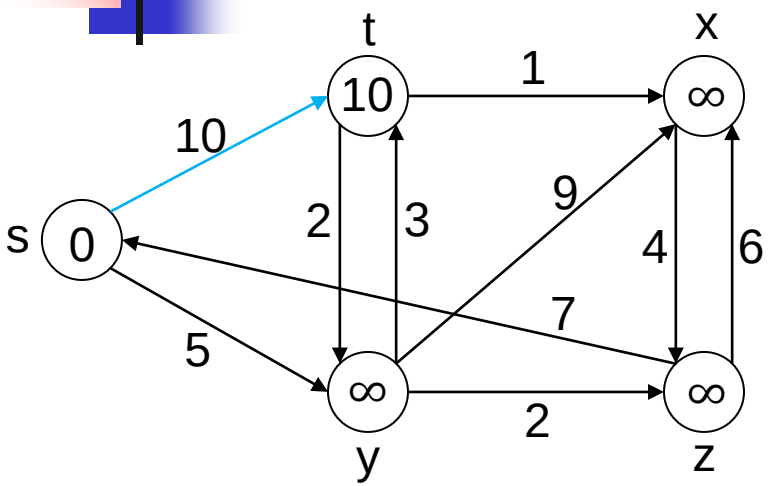

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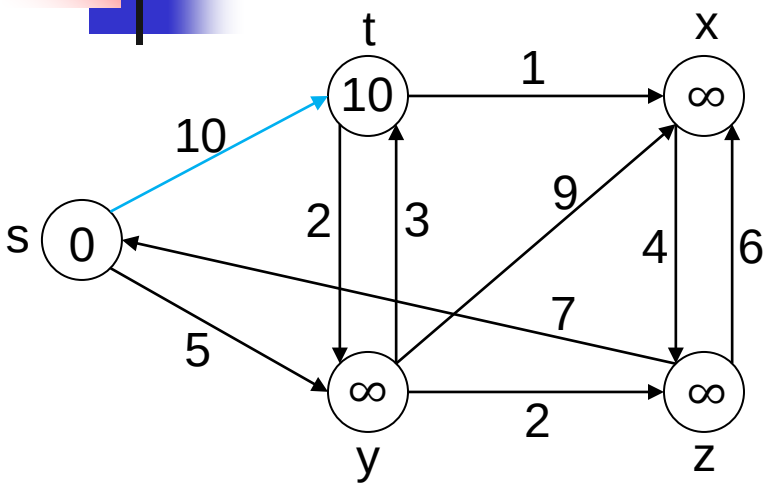

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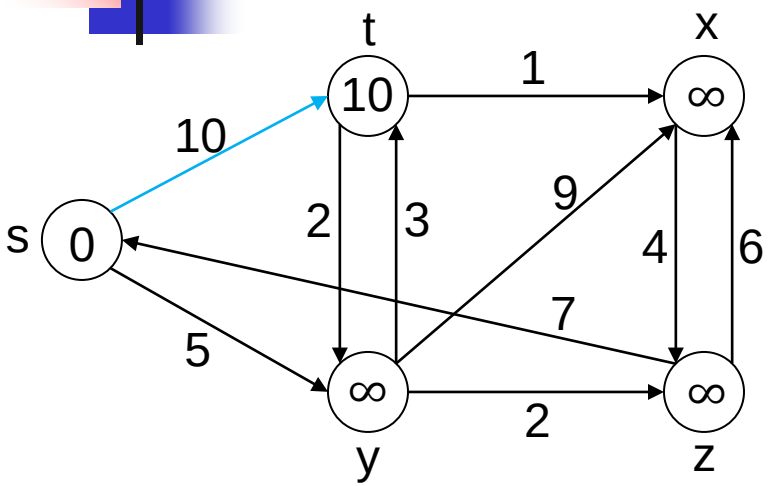

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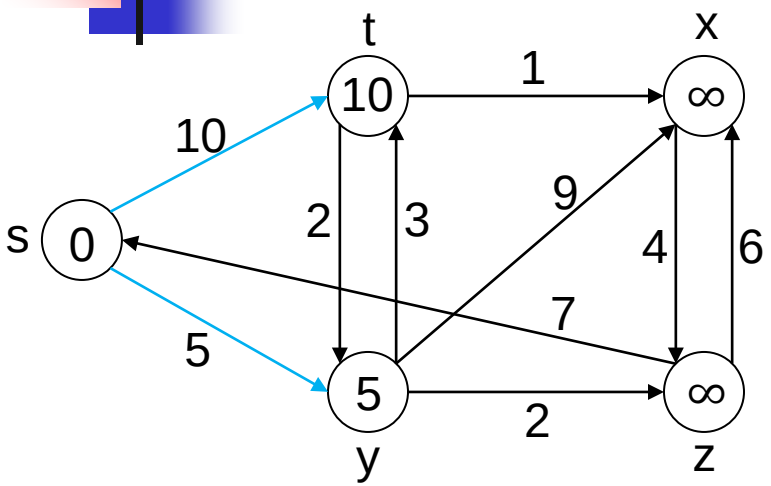

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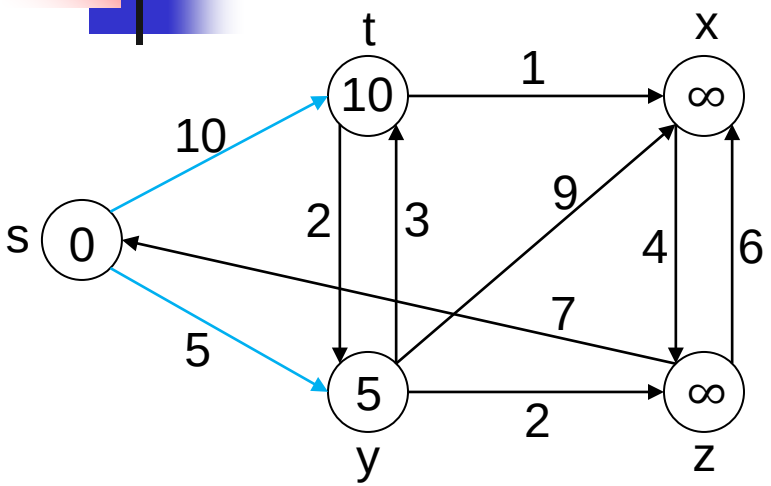

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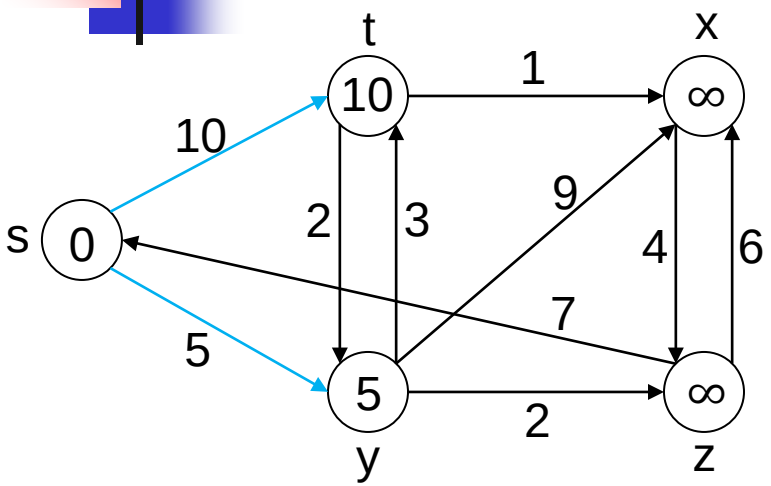

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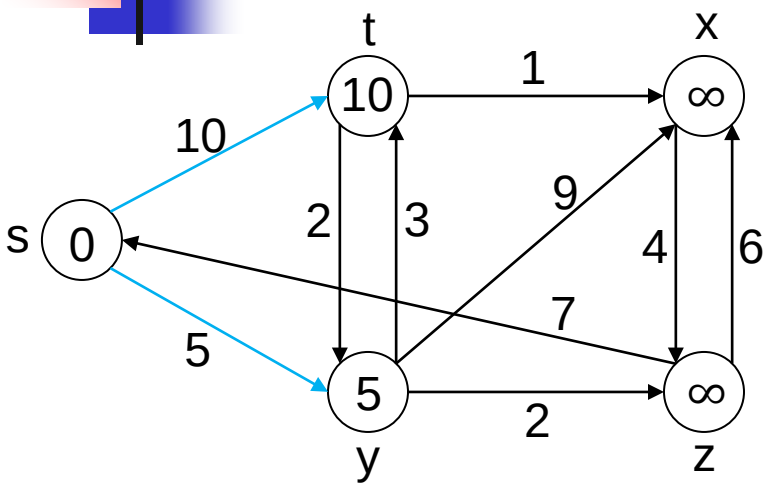

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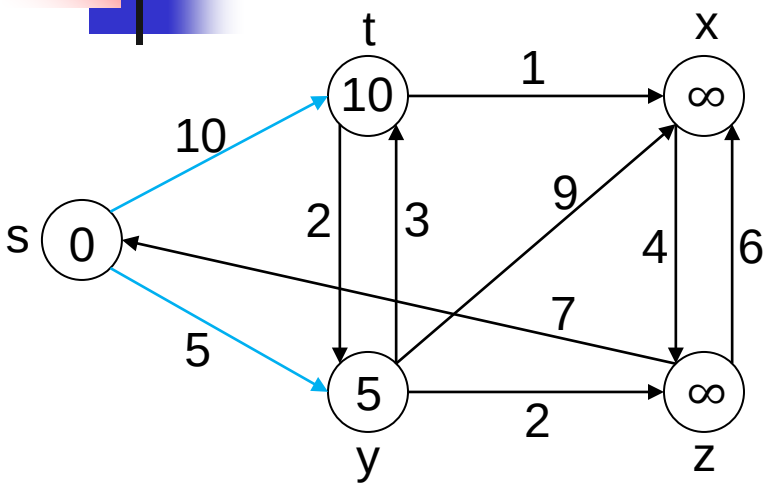

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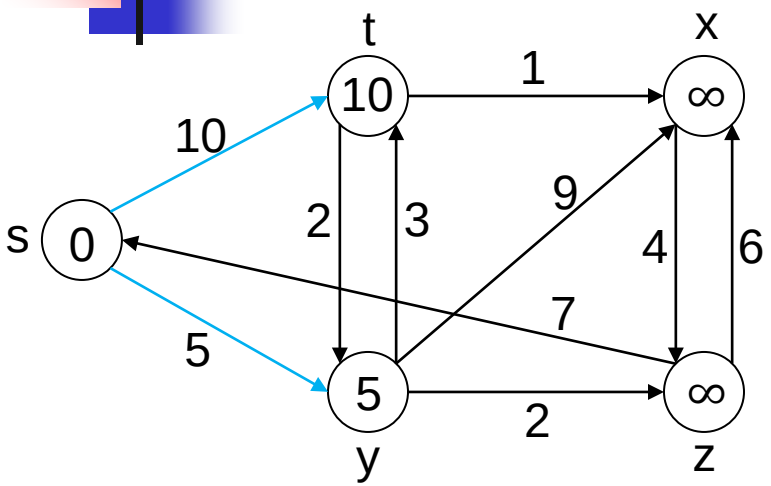

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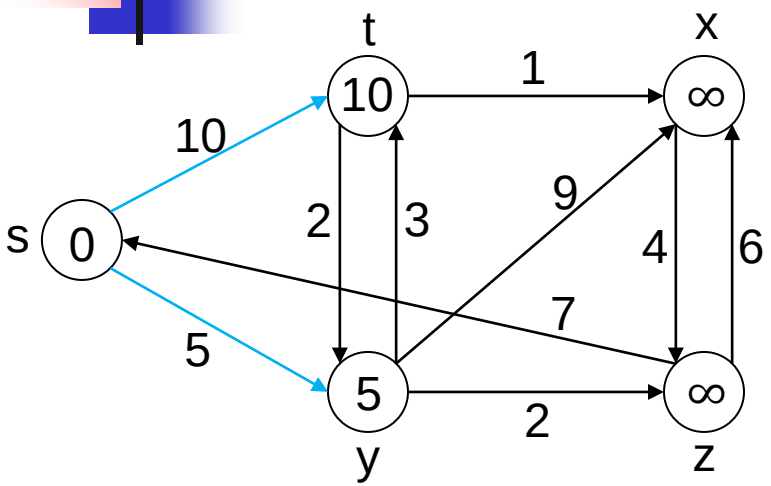

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alt = 5 + 3 = 8

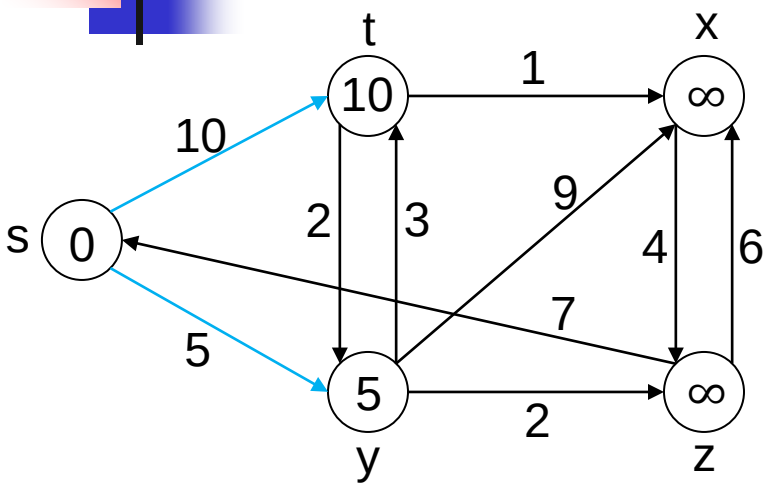
```
Dijkstra (G, s)
    for each vertex v in G
        dist[v] = ∞
        previous[v] = Undefined
    dist[s] = 0
    Q = G.V
    while Q is not empty
        u = node in Q with smallest dist[]
        remove u from Q
        for each neighbor v of u
            alt = dist[u] + dist_between(u, v)
            if alt < dist[v]
                dist[v] = alt
                previous[v] = u
    return previous[]
```

dist[v]: shortest distance from ***s*** to ***v***

previous[v]: previous vertex in shortest path to v

Q: set of vertices

Dijkstra Algorith



Q:[s, t, y, x, z]

u = y

v = t

alt = 5 + 3 = 8 < 10

```
Dijkstra (G, s)
    for each vertex v in G
        dist[v] = ∞
        previous[v] = Undefined
    dist[s] = 0
    Q = G.V
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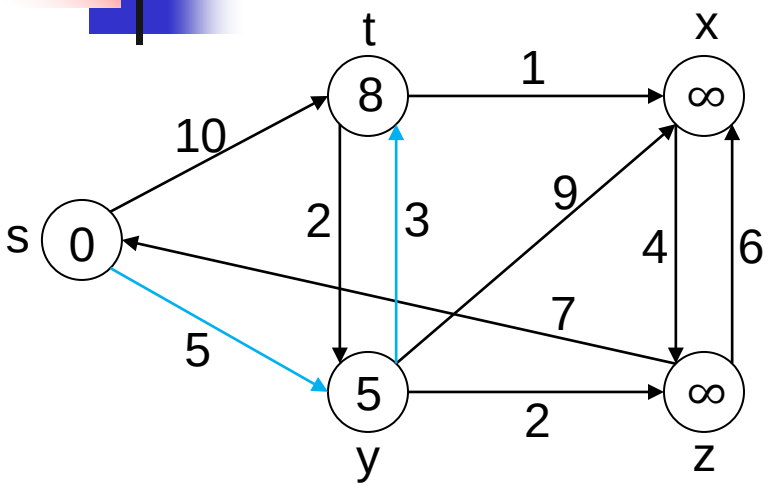
Q: set of vertices

Dijkstra Algorith



Q:[s, t, y, x, z]

u = y

v = t

alt = 5 + 3 = 8 < 10

```
Dijkstra (G, s)
    for each vertex v in G
        dist[v] = ∞
        previous[v] = Undefined
    dist[s] = 0
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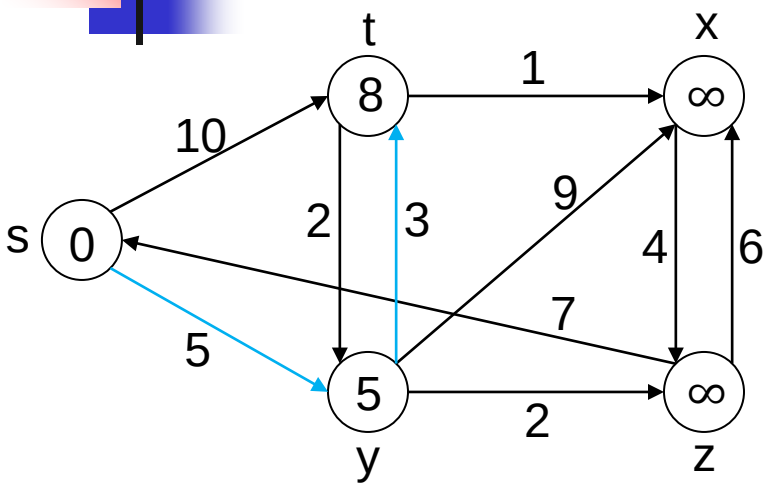
Dijkstra Algorith



Q:[~~s~~, t, ~~y~~, x, z]

u = y

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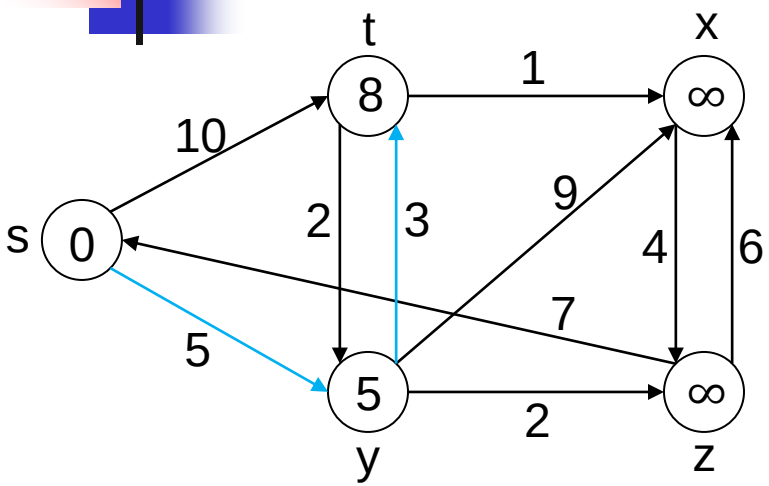
Dijkstra Algorith



Q:[~~s~~, t, ~~y~~, x, z]

u = y

v = x

alt = 5 + 9 = 14

```
Dijkstra (G, s)
    for each vertex v in G
        dist[v] = ∞
        previous[v] = Undefined
    dist[s] = 0
    Q = G.V
    while Q is not empty
        u = node in Q with smallest dist[]
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    return previous[]
```

dist[v]: shortest distance from ***s*** to ***v***

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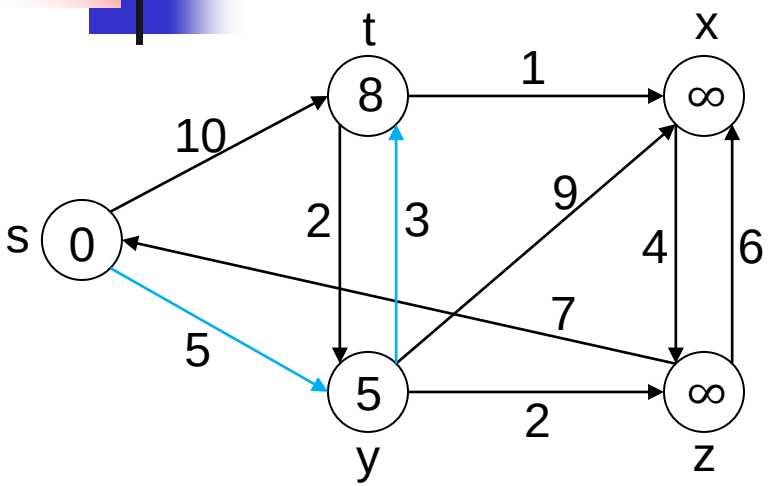
Q: set of vertices

Dijkstra Algorith



Q:[~~s~~, t, ~~y~~, x, z]

u = y

v = x

alt = 5 + 9 = 14 < ∞

```
Dijkstra (G, s)
    for each vertex v in G
        dist[v] = ∞
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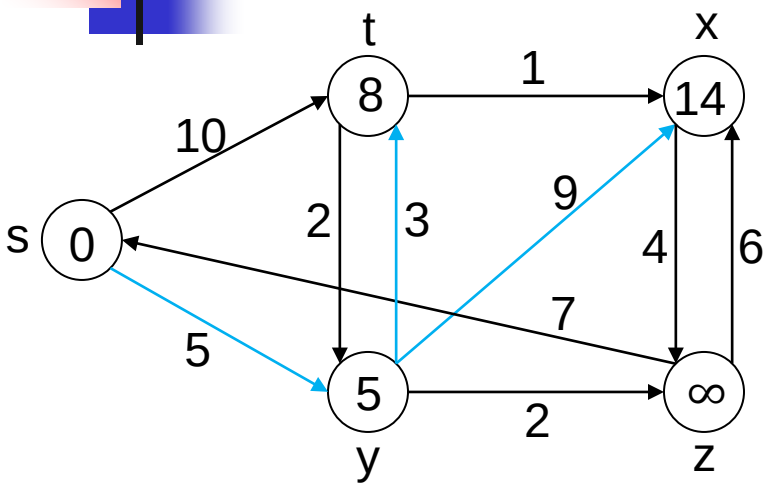
Dijkstra Algorith



Q:[~~s~~, t, ~~y~~, x, z]

u = y

v = x

alt = 5 + 9 = 14 < ∞

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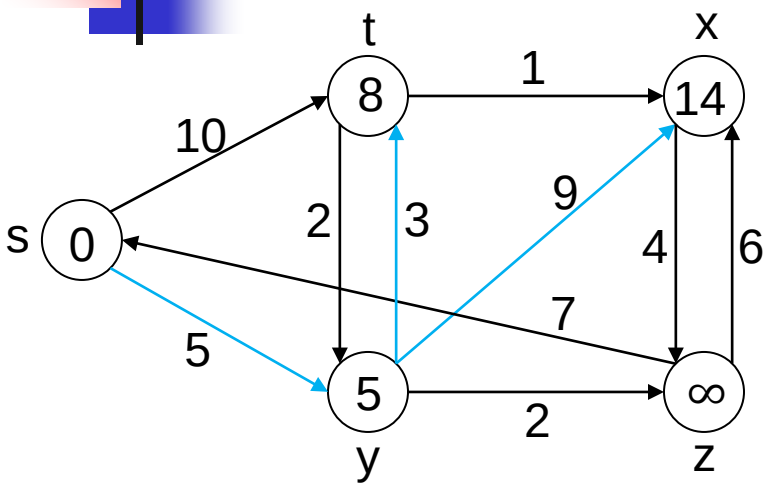
Dijkstra Algorith



Q:[~~s~~, t, ~~y~~, x, z]

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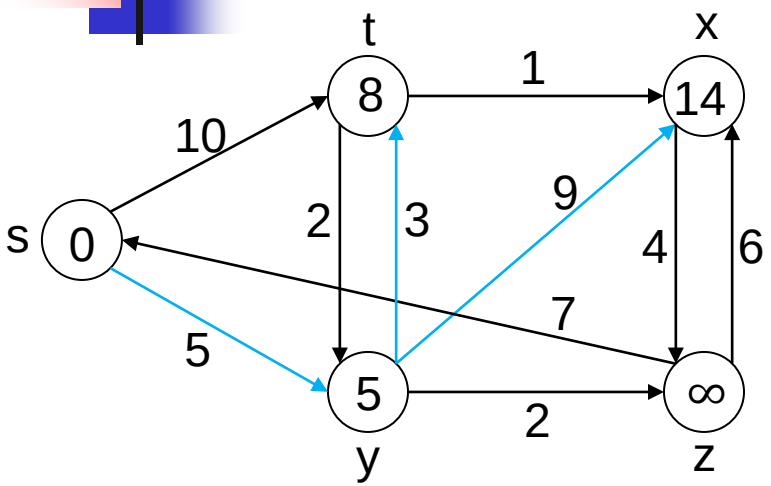
Dijkstra Algorith



Q:[s, t, y, x, z]

u = y

v = z

alt = 5 + 2 = 7

```
Dijkstra (G, s)
    for each vertex v in G
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```

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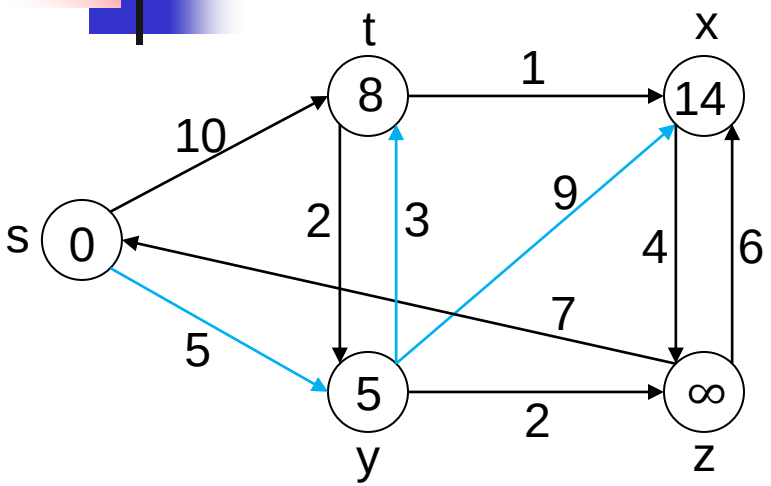
Q: set of vertices

Dijkstra Algorith



Q:[~~s~~, t, ~~y~~, x, z]

u = y

v = z

alt = 5 + 2 = 7 < ∞

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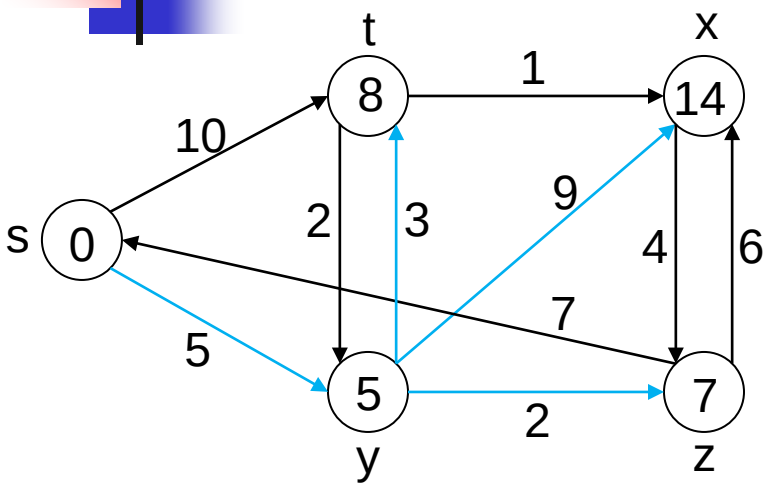
Q: set of vertices

Dijkstra Algorith



Q:[~~s~~, t, ~~y~~, x, z]

u = y

v = z

alt = 5 + 2 = 7 < ∞

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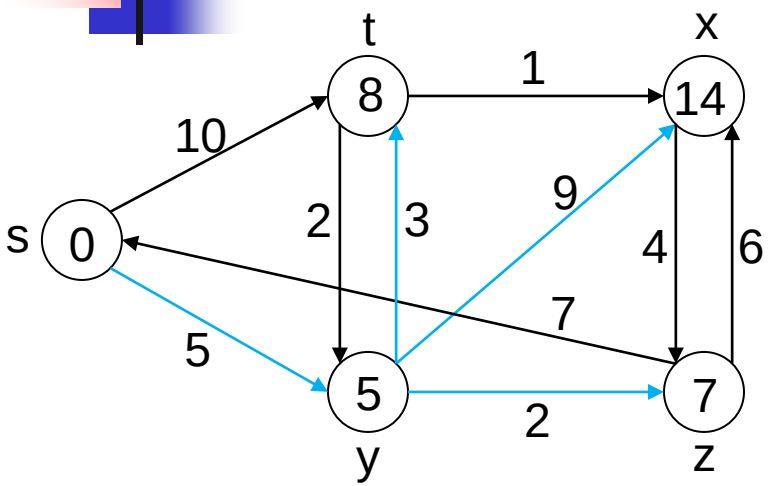
Dijkstra Algorith



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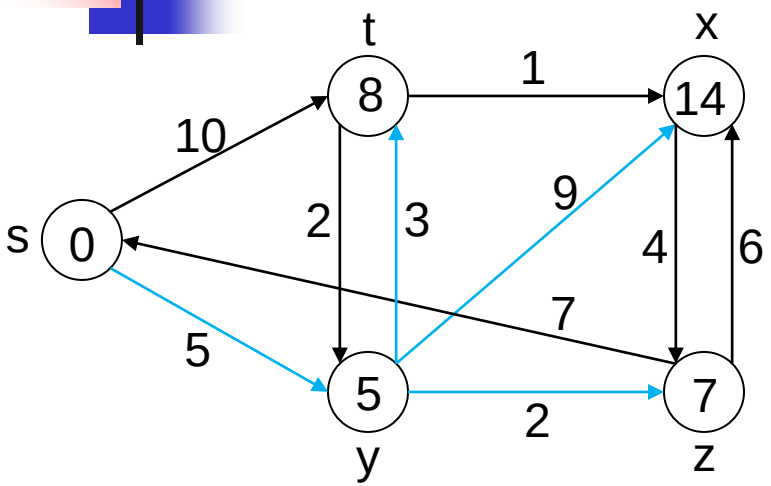
Dijkstra Algorith



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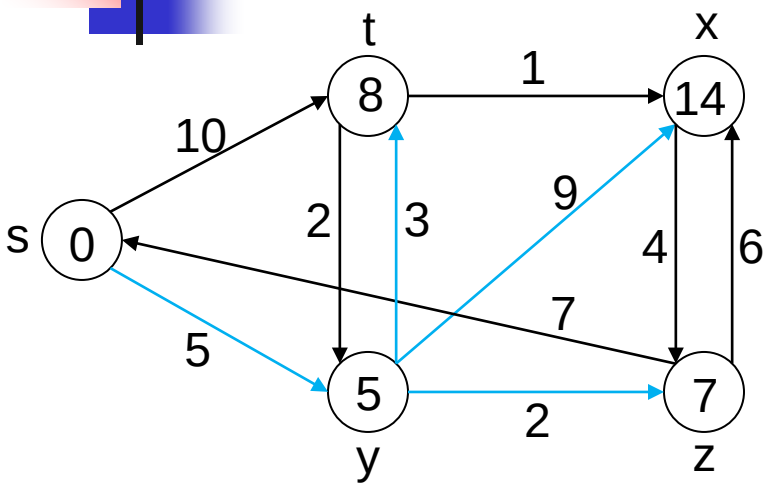
Dijkstra Algorith



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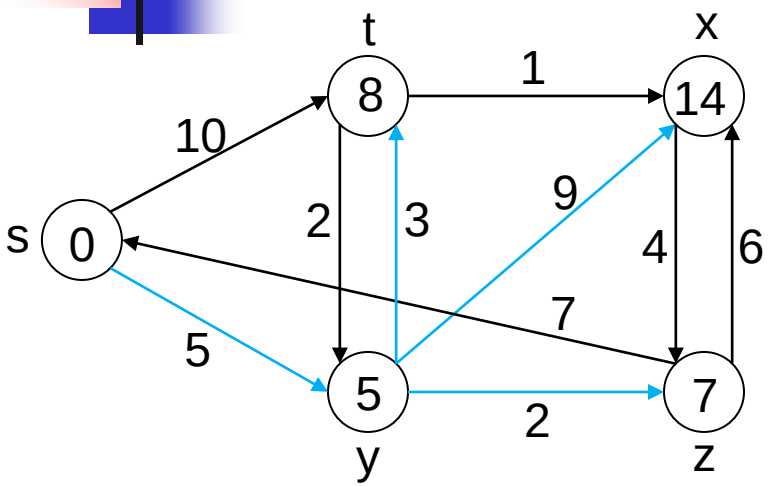
Dijkstra Algorith



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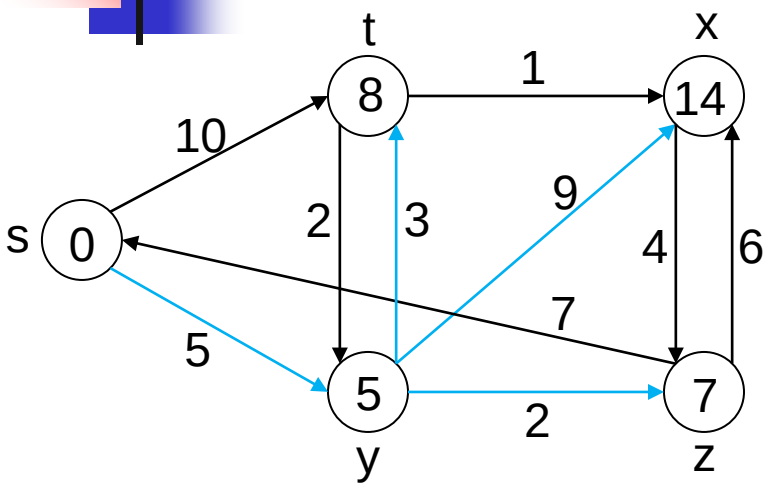
Dijkstra Algorith



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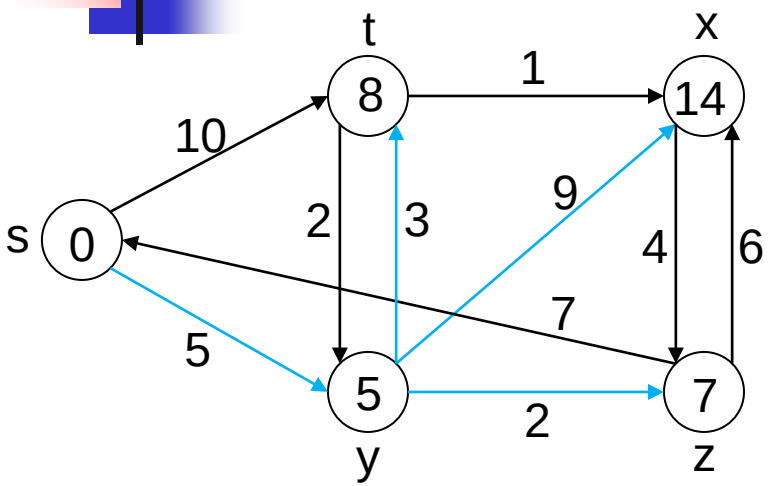
Dijkstra Algorith



Q:[~~s~~, t, ~~y~~, x, ~~z~~]

u = z

v = s

alt = 7 + 7 = 14

```
Dijkstra (G, s)
    for each vertex v in G
        dist[v] = ∞
        previous[v] = Undefined
    dist[s] = 0
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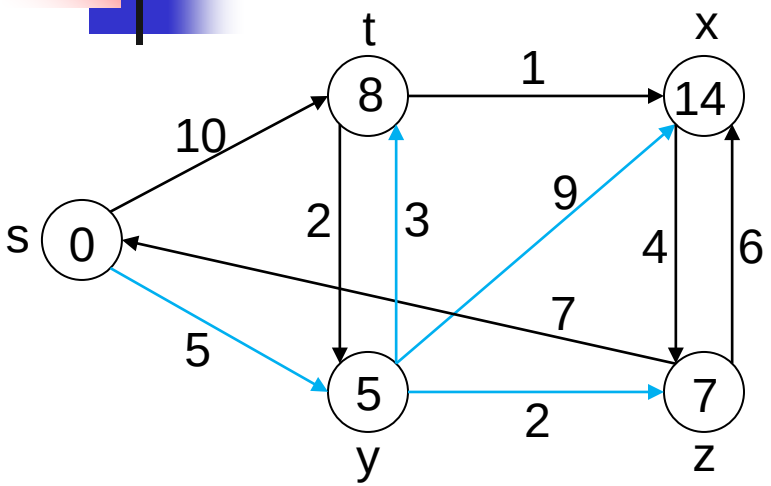
Dijkstra Algorith



Q:[~~s~~, t, ~~y~~, x, ~~z~~]

u = z

v = s

alt = 7 + 7 = 14 > 0

```
Dijkstra (G, s)
    for each vertex v in G
        dist[v] = ∞
        previous[v] = Undefined
    dist[s] = 0
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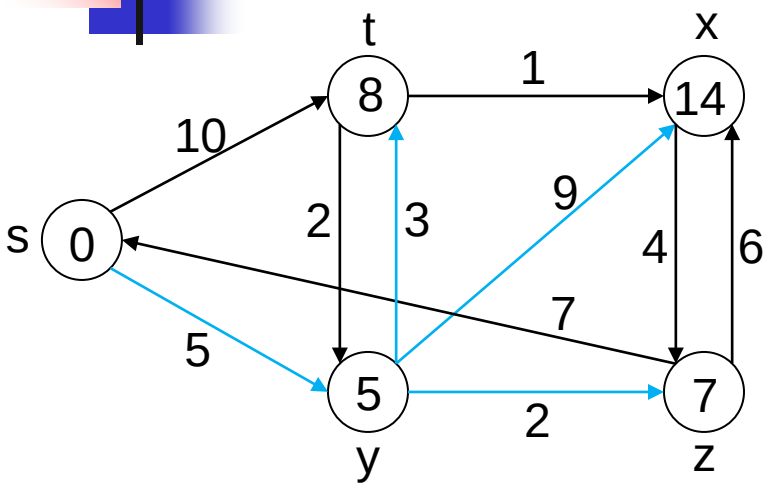
Dijkstra Algorith



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dist[v]: shortest distance from ***s*** to ***v***

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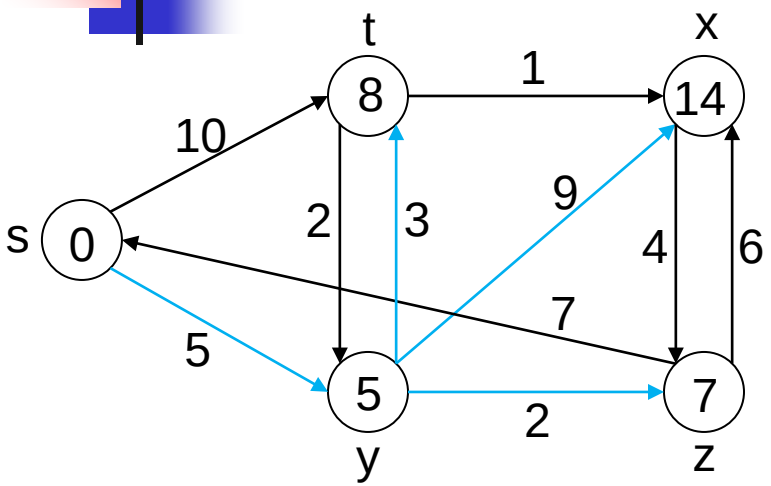
Dijkstra Algorith



Q:[~~s~~, t, ~~y~~, x, ~~z~~]

u = z

v = x

alt = 7 + 6 = 13

```
Dijkstra (G, s)
    for each vertex v in G
        dist[v] = ∞
        previous[v] = Undefined
    dist[s] = 0
    Q = G.V
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dist[v]: shortest distance from ***s*** to ***v***

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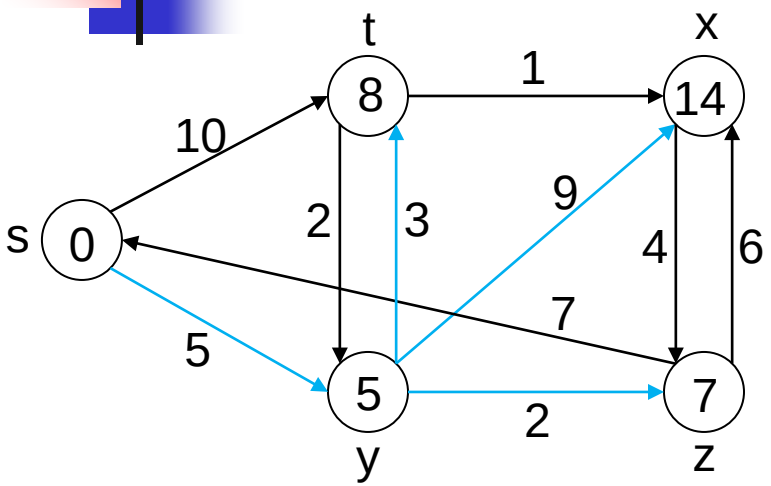
Dijkstra Algorith



Q:[~~s~~, t, ~~y~~, x, ~~z~~]

u = z

v = x

alt = 7 + 6 = 13 < 14

```
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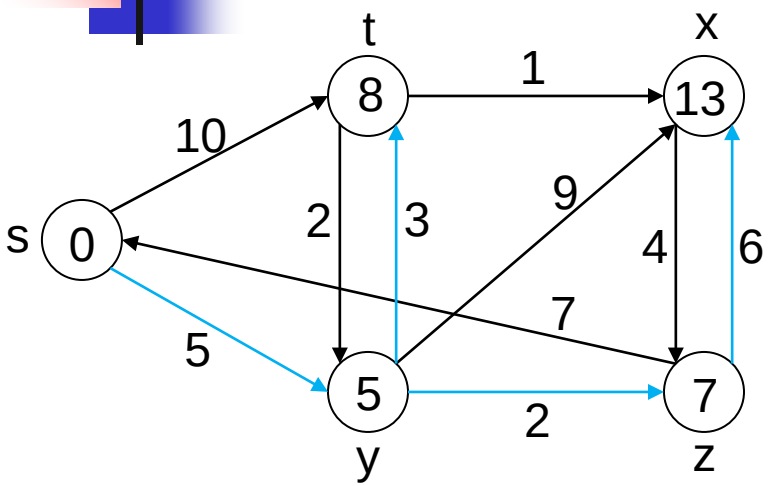
Dijkstra Algorith



Q:[~~s~~, t, ~~y~~, x, ~~z~~]

u = z

v = x

alt = 7 + 6 = 13 < 14

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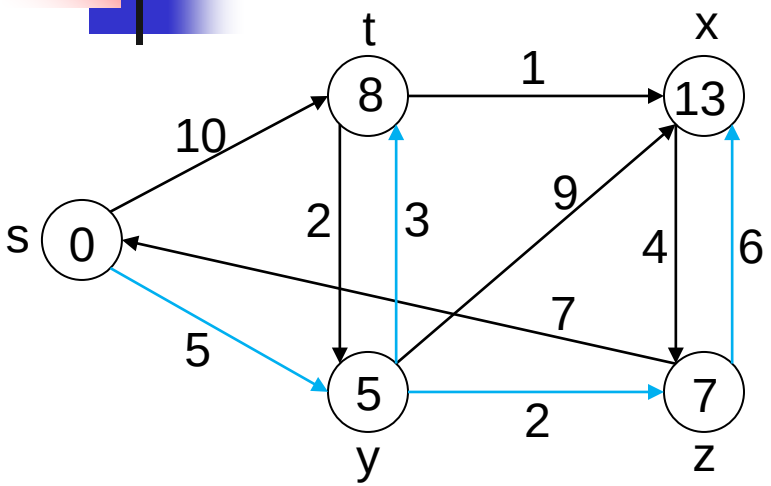
Dijkstra Algorith



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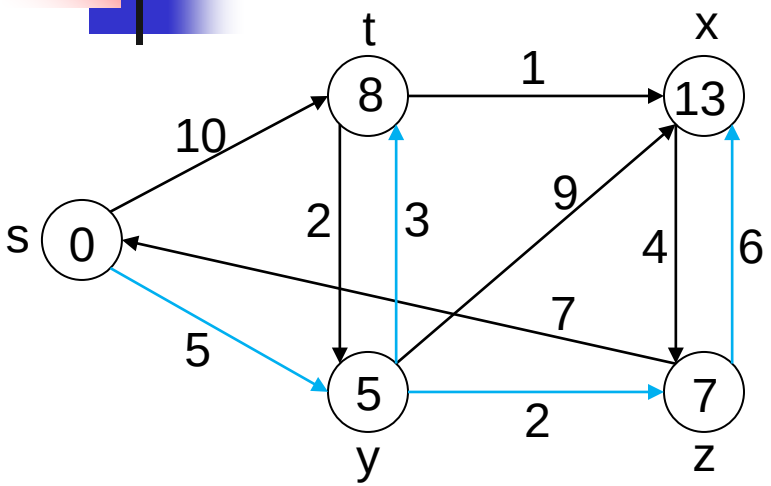
Dijkstra Algorith



Q:[~~s~~, t, ~~y~~, x, ~~z~~]

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Dijkstra (G, s)
    for each vertex v in G
        dist[v] = ∞
        previous[v] = Undefined
    dist[s] = 0
    Q = G.V
    while Q is not empty
        u = node in Q with smallest dist[]
        remove u from Q
        for each neighbor v of u
            alt = dist[u] + dist_between(u, v)
            if alt < dist[v]
                dist[v] = alt
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    return previous[]
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dist[v]: shortest distance from ***s*** to ***v***

previous[v]: previous vertex in shortest path to v

Q: set of vertices

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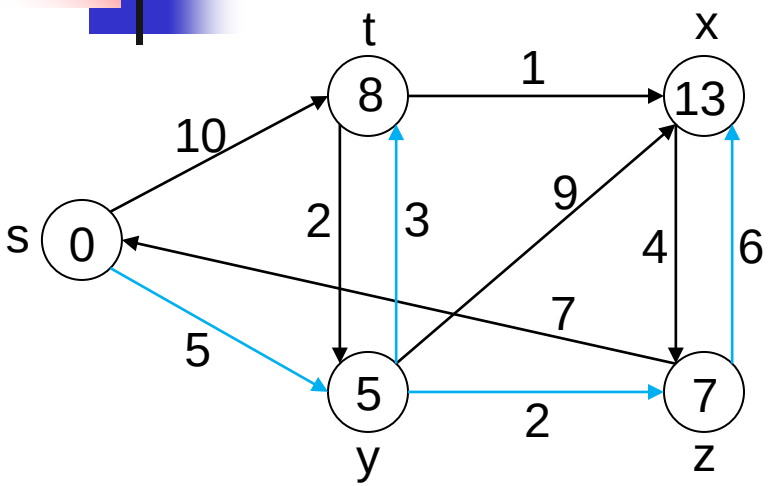

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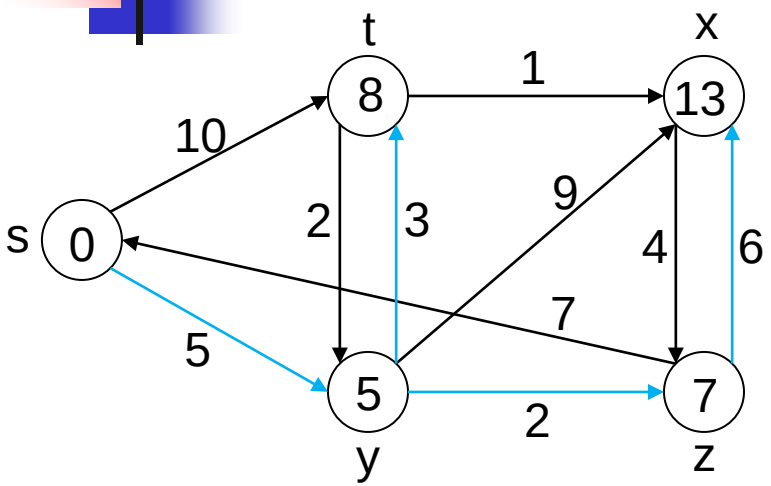

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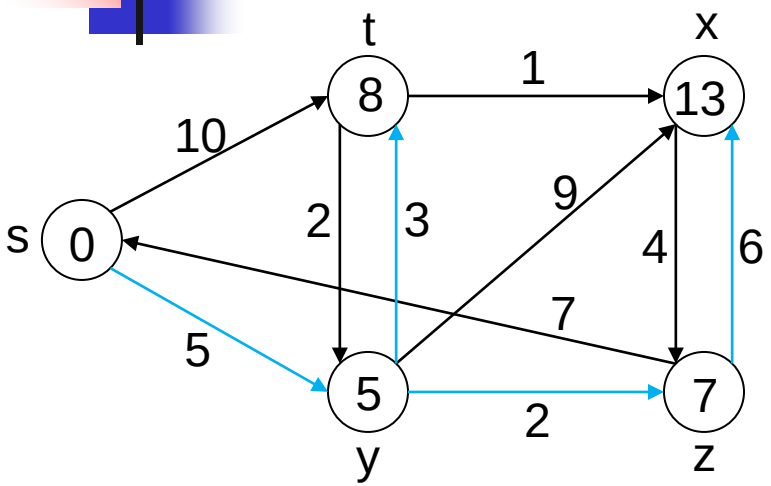

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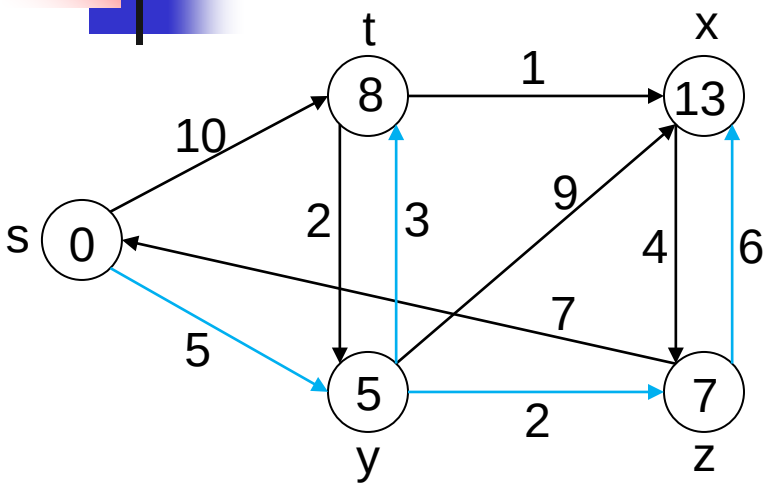

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u = t

v = y

alt = 8 + 2 = 10

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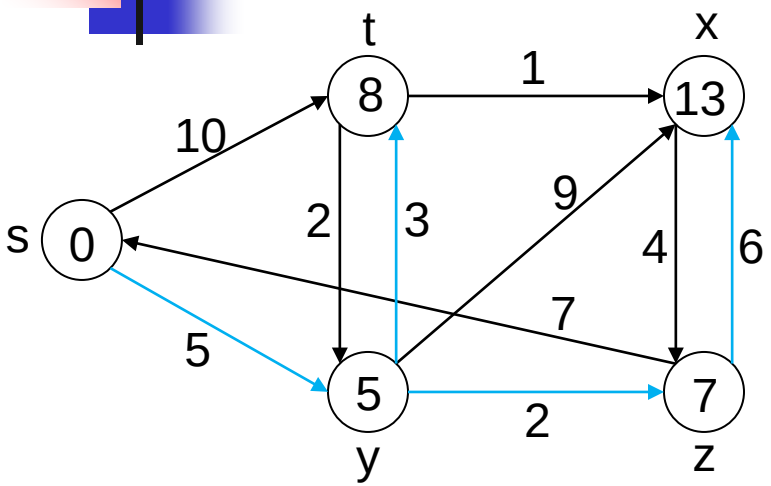

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v = y

alt = 8 + 2 = 10 > 5

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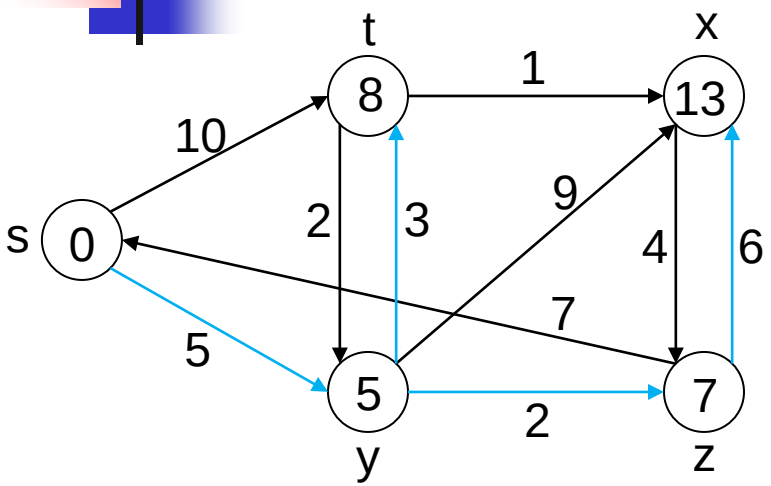

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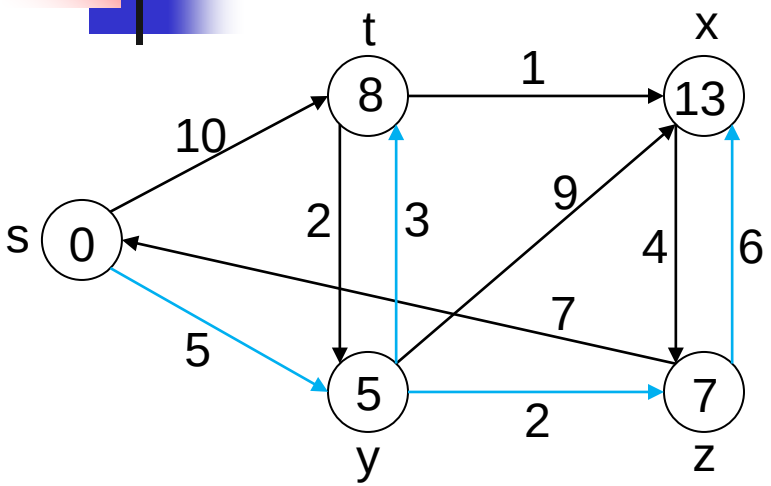

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v = x

alt = 8 + 1 = 9

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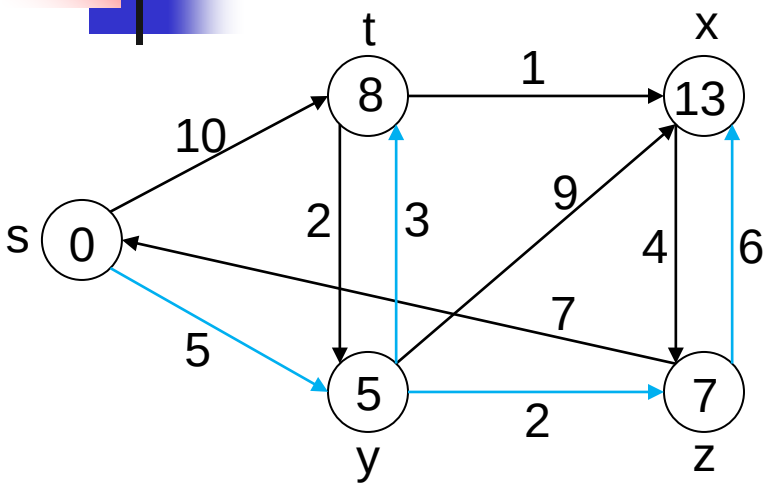

Q:[~~s~~, ~~t~~, ~~y~~, x, ~~z~~]

u = t

v = x

alt = 8 + 1 = 9 < 13

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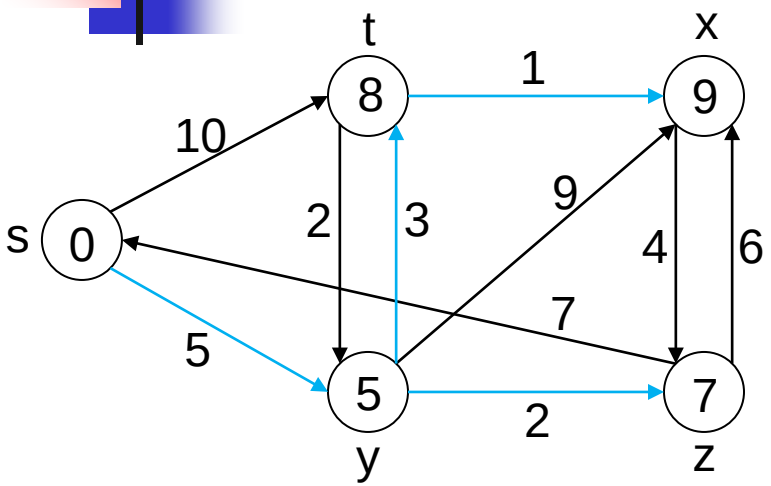

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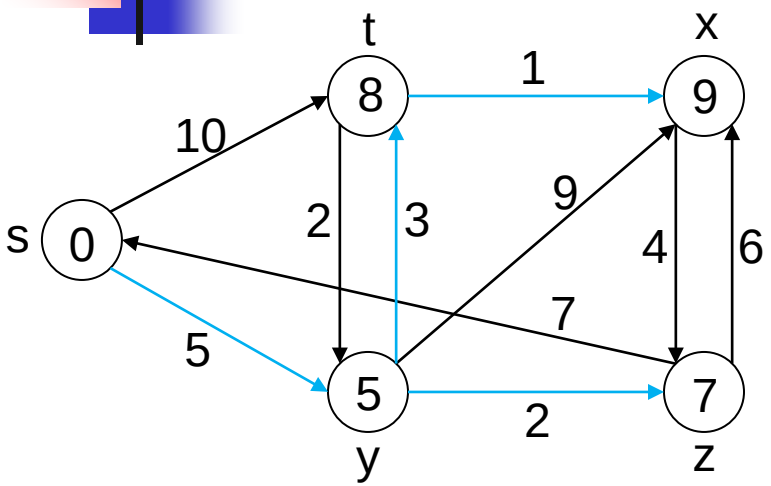

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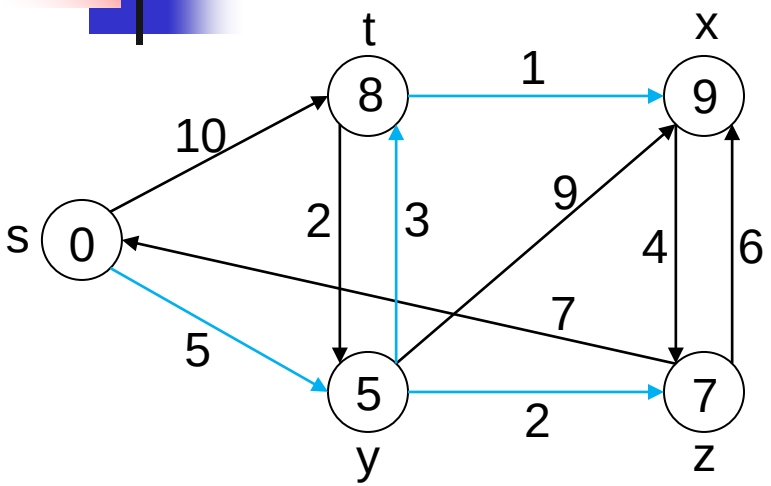

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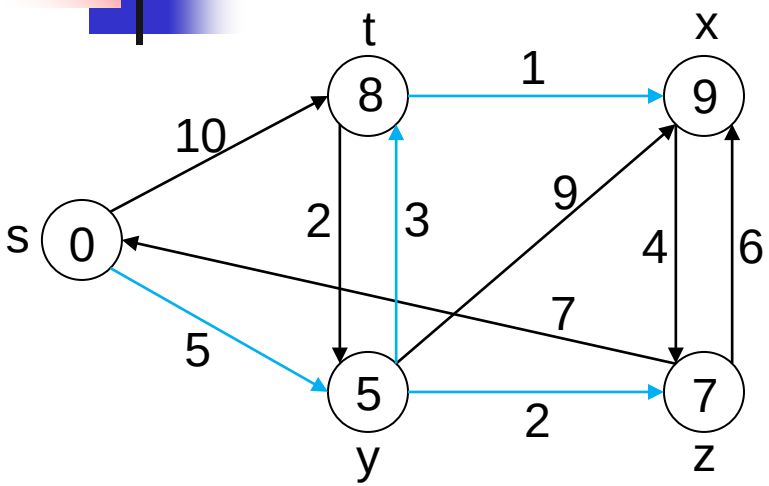


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